A
TREATISE
OF
GAUGING.

CONTAINING

Not only what is common on the Subject, but likewiſe a great Variety of new and intereſting IMPROVEMENTS.

WITH THE

DEMONSTRATIONS of ſeveral very uſeful and remarkable Properties of VESSELS and INSTRUMENTS, relative to this Art.

Illuſtrated with neceſſary EXAMPLES, and adapted both to the ſpeculative and practical READERS.

By THOMAS MOSS.

LONDON:
Printed for the AUTHOR, and ſold at his Houſe
in *Roe-buck Court*, *Chiſwell-ſtreet*: Alſo by W.
OWEN, near *Temple-Bar*, *Fleet-ſtreet*; Z. STUART,
at the *Lamb*, in *Pater-noſter Row*; and J. JOHNSON, oppoſite the *Monument*.

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SON, oppoſite the Monument. [illegible]

TO THE

HONOURABLE

THE

COMMISSIONERS OF HIS MAJESTY'S REVENUE OF EXCISE, WITHIN THE KINGDOM OF *ENGLAND*, *&c.*

THE FOLLOWING

TREATISE,

Containing ſuch IMPROVEMENTS as will, it is humbly preſumed, contribute to the Service of the REVENUE, and alſo be of real Advantage to the practical GAUGER,

IS

MOST HUMBLY DEDICATED,

By their HONOURS *Faithful,*

and moſt Obedient Servant,

Thomas Moſs.

THE

PREFACE.

THERE are ſo many different Compoſitions already extant on the Subject of Gauging, that Some, perhaps, may imagine no farther Improvements can poſſibly be made therein; but I flatter myſelf, however, that the attentive *and* unprejudiced *Reader will find ſeveral uſeful Things, in the following Sheets, not to be met with in any other Treatiſe on the Subject.*

In the Courſe of this Undertaking, I have exerted my utmoſt Endeavours to extend *the* ART *of* GAUGING; *by laying down the moſt general, accurate, and eaſy Methods of determining the Meaſures of all the various Forms of Veſſels which occur in Practice: In what Manner my Deſign is executed, is wholly ſubmitted to the* impartial *Reader.*

I have been ſtudiouſly anxious to promote Truth and Utility, without even attempting to depreciate the Labours of Others; as judging it more commendable to paſs over any little Imperfection that occurred, than to endeavour to magnify it, with a View to enhance the Merit of my own Performance: And although I have, in many Inſtances, departed from former Writers on the Subject, the greateſt Care has been taken not to introduce any new *Methods of Gauging, but ſuch as will, it is preſumed, be found, by the Practitioner, to be far more general, and not more difficult, than thoſe which are omitted; and alſo ſuch as are founded upon the moſt indubitable Principles: Which Principles, to oblige the inquiſitive Reader, are given in the* Notes, *and it is hoped they will not be decried by thoſe Gentlemen* *who may want Time, or Inclination, to apply themſelves to the ſpeculative Part of the Subject.*

Though it muſt be allowed, that there is no Neceſſity for the practical Gauger to be acquainted with Geometry, Algebra, *and* Fluxions; *yet I can with Safety affirm (what has been already remarked, by that excellent Mathematician, Mr.* Robert Shirtcliffe, *in his Theory and Practice of Gauging)* "*That without a competent Skill in Algebra and Geometry, it is abſolutely impoſſible for any Perſon to determine, whether the Rules given by common Writers upon this Subject be true or falſe, and much leſs make any (even the leaſt) Improvement in this Part of Science:*" — *And therefore, I cannot but think it extremely abſurd, and ill-natured in any* One, *to endeavour to depreciate, and, what is very aſtoniſhing, even to ridicule thoſe excellent Branches of Science, from whence are derived the very Rules and Inſtruments, which are ſo highly approved by every practical Gauger; without which, he muſt acknowledge, even his daily Buſineſs could not be performed.*

As it would be unneceſſary, in this Place, to enumerate all *the Particulars that compoſe the enſuing Sheets; it may therefore ſuffice to point out only* a few *of thoſe Articles, which, perhaps, the candid and practical Reader will look upon as real and uſeful Improvements.*

In the Buſineſs of Caſk-Gauging (which is reckoned the moſt difficult Part of the Subject) is given a general and practical Method of determining, very nearly, *the* true *Variety of any cloſe Caſk; whereby any Perſon, with a very little Application, may be enabled to form a tolerably good Idea of the Variety,* by Inſpection only: *This, it is preſumed, is an Improvement, which, if duly attended to, will be found of ſingular Advantage; ſince it will, doubtleſs, be a*

Means of preventing ſuch Errors as muſt unavoidably happen, by the ordinary Methods of merely gueſſing *at the Variety of the Caſk. — In this Branch of Gauging are alſo given, two very eaſy and comprehenſive Methods of finding the true Mean-Diameters of the three different Varieties of Caſks, let the Proportion of the Bung and Head-Diameters be what it will: For on ſuch Proportion (and not upon the Difference of thoſe Diameters) the* true *Multiplier, for finding a Mean-Diameter, wholly depends.*

The Nature and Property of the Diagonal Rod are far more extenſively conſidered than heretofore; with very plain and uſeful Directions for applying this Inſtrument, with Certainty, to a great Variety of different Forms of Caſks. — It has been hitherto imagined, that the Diagonal Rod would only exhibit the true *Contents of* one *particular Form of Caſks; and alſo that its original Conſtruction was from a Caſk, whoſe Diagonal is* 30 *Inches, and Content* 60 *Ale Gallons, or from ſome known Content and its correſponding Diagonal, as they appear on Gauging Rules. — That the Diagonal of a Caſk may be* 30 *Inches, and its Content* 60 *Ale Gallons (or about* $73\frac{1}{4}$ *Wine Gallons) is indiſputably evident: But it certainly does not follow from thence, that there can be but* one *Bung-Diameter, Head-Diameter, and Length, allotted for a Caſk, which can have the above-mentioned Diagonal and Content; becauſe* all *thoſe Dimenſions, and conſequently the Form of the Caſk, may vary; without altering either its* Diagonal, Magnitude, *or* Variety: *See* Sect. X. Pa. 194.

The Methods of ullaging both ſtanding and lying Caſks, by the Pen, are given in as plain and conciſe a Manner as poſſible; with very eaſy Directions for determining when the Lines of Segments on the Sliding-Rule may be depended on, and alſo whether the Error is in Exceſs or Defect.

PREFACE.

The Method of approximating the Meaſure of any curvilineal Plane, by Means of equidiſtant perpendicular Ordinates (or Diameters), is delivered with as much Perſpicuity and Conciſeneſs, as the Nature of ſo important a Subject will poſſibly admit of; and which is moreover illuſtrated with proper Examples, not only of Figures whoſe Properties are known, and the Areas thereof determinable by other Methods; but alſo of Figures whoſe Properties are unknown, and their Areas not to be determined, with any Certainty, by any other Method whatever.

Very accurate Tables are given of the Areas of Circles in Ale and Wine Gallons, each to 120 *Inches Diameter. — Tables of this Nature are indeed to be met with in moſt Authors on this Subject; but, however, the Methods of Computation (being more exact and eaſy than any that have occurred to me), by which the Tables in* Pa. 240, &c. *were actually formed, will not, it is apprehended, be unacceptable to ſuch Perſons as may be deſirous of extending the ſaid Tables to larger Diameters.*

Many other uſeful and intereſting Particulars might here be mentioned, but I rather chooſe to refer to the Work itſelf; and therefore ſhall only beg, that the Reader will not too haſtily *cenſure and condemn it; but that, after impartially peruſing it with proper Attention, he will candidly excuſe ſuch Defects as may occur to him, and have eſcaped my Obſervation: This, it is hoped, is no unreaſonable Requeſt; ſince it is but ſolliciting that Indulgence which every One is intitled to, who lays his* own *Sentiments before the Public, without ſhewing too high an Opinion of his own Abilities.*

Roe-buck Court, Chiſwell-ſteeet,
January 16. 1766.

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ERRATA.

Page 24, Cor. 1, Line 1, for *consist*, read *consists*.

P. 25, dele *its Square Root will have three Places*, &c. Cor. 2, L. 4, for *either will be*, r. *will be either*.

P. 34, L. 8, for *wholly*, r. *chiefly*.

P. 45, L. 5 and 8 from the Bottom, for *is*, r. *in*.

P. 64, L. 5, in the *Note*, for ACE, r. ACF.

P. 65, Prop. 3, L. last but one, after Bm place a *Comma*. L. 5 and 6 in the *Note*, for 4BM², r. 4EM².

P. 91, L. last but one, for *ce*, r. *cr*.

P. 92, L. 5, in the *Note*, for GDCB, r. ODCB, and L. 9, for *ce*², r. *ae*.

P. 113, L. 6, in the *Note*, for AacbBA, r. AacbBCA.

P. 124, for $\left.\overline{\frac{Ap-Ax+ax}{p}}\right|^{2}$ r. $\left.\overline{\frac{Ap-Ax+ax}{p}}\right|^{2}$.

P. 136, in the Cor. for *is* CD *and the Height* CE, r. *and Height each* = CD.

P. 166, L. 17, in the *Note*, for AB, r. AD.

P. 182, *Tab.* 1, against .56, for .878, r. .8781, against .80, for .9388, r. .9380, and opposite .90, for .9611, r. .9671.

P. 225, for *Example* 1, r. *Example* 2.

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BRITISH MUSEUM

A TREATISE OF GAUGING.

SECTION I.

Of DECIMAL FRACTIONS.

IT is indiſpenſably required of every One, who would learn the Art of Gauging, to acquire a previous and competent Knowledge of Decimal Fractions; as the Dimenſions of all Utenſils, of what Form ſoever, are taken in Inches and Tenths, all Inſtruments, for that Purpoſe, being decimally divided: I therefore apprehend it will not be improper to give, by way of Introduction, a ſuccinct Account of Decimal Fractions.

Firſt then, in Order to form as clear an Idea as poſſible of the Nature of Fractions in general, let us conceive an Unit, Integer, or *one* whole Thing, of what Denomination ſoever, whether it be Coin, Weight, Meaſure or Time, *&c.* to be divided into a certain Number of equal Parts; then this Number of equal Parts, be it what it will, is ever called the Denominator of the Fraction; and that Number ſhewing how many of theſe Parts are to be taken, or expreſſed, is called the Numerator.

Thus, for Example, ſuppoſe an Unit, or Integer, to be divided into 12 equal Parts, and that it was required to expreſs 5 of thoſe Parts in a Vulgar Fraction; then 12 will be the Denominator, and 5 the Numerator, and the Fraction itſelf, will, by writing the Numerator above the Denominator, with a Line betwixt them, be thus expreſſed $\frac{5}{12}$, and is read Five-twelfths:——This is a general Notation for Fractions of all Denominations.

But, in Decimal Fractions, where the *Integer*, or *Whole Thing*, is ſuppoſed to be divided into 10, 100, or 1000, *&c.* equal Parts, the Notation will be more commodious for Practice, by writing down the Number of Parts to be taken with a Point, or Comma, prefixed, without putting down the Denominator, as in Vulgar Fractions; it being abſolutely unneceſſary here, ſince it is always known to be an Unit, with as many Cyphers annexed, as there are Places in the Fraction taken. Thus $\frac{5}{10}$ (Five-tenths) expreſſed decimally, will be .5, and $\frac{75}{100}$ (Seventy-five Hundredths) will be .75.

It will be proper to obſerve, to the Learner, that Cyphers placed on the left Hand of a Decimal Fraction, decreaſe its Value in a ten-fold Proportion, in the ſame Manner as Cyphers placed on the right Hand of a whole Number, increaſe the Value thereof.

For Example, .5 ($\frac{5}{10}$) is the Half of an Unit; but .05 ($\frac{5}{100}$) is only $\frac{1}{20}$th of an Unit, which is

$\frac{1}{10}$th

BRITISH MUSEUM

$\frac{1}{10}$th of the former; alſo .005 ($\frac{5}{1000}$) is only $\frac{1}{200}$th Part of an Unit, and is therefore $\frac{1}{10}$th of the laſt Fraction .05, *&c.*

It may alſo be proper to take Notice, that Cyphers placed on the right Hand of a Decimal Fraction, neither augment nor diminiſh its Value. For .5 (or Five-tenths) of any Thing, are the very ſame in Value as .50 (or Fifty-hundredths) of the ſame Thing: In the former of theſe, the Integer is ſuppoſed to be divided into 10 equal Parts, and in the latter into 100 equal Parts; hence it is very obvious, that 5 of the 10 equal Parts are equivalent to 50 of the 100 equal Parts, of the ſame *Integer*, or *Unit*.

	Hundreds of Thouſands.	*Tens of Thouſands.*	*Thouſands.*	*Hundreds.*	*Tens.*	*Unit's Place.*	*Tenths.*	*Hundredths.*	*Thouſandths*	*Ten Thouſandths.*	*Hundred Thouſandths.*	
&c.	5	4	3	2	1	0.	1	2	3	4	5	&c.

In the preceding Table, it is very obvious that the Figures on the left Hand of the Point are Integers, or whole Numbers, and thoſe on the right Hand are called Decimal Fractions.

PROP. I.

To reduce any given Vulgar Fraction, into a Decimal Fraction.

RULE.

Let a competent Number of Cyphers be annexed to the Numerator, to form a Dividend; which being divided by the Denominator, the Quotient (if there happens to be no Remainder) will be *preciſely* the Decimal Fraction ſought.*

EXAMPLES.

How muſt $\frac{3}{4}$ and $\frac{7}{8}$ of an Unit, or Integer, be expreſſed in Decimal Fractions?

OPERATIONS.

```
4)3.00(.75        8)7.000(.875
  28                64
 ----              ----
   20                60
   20                56
 ----              ----
   ..                 40
 ----                 40
                     ----
                      ..
                     ----
```

Hence

---

* A Vulgar Fraction cannot *preciſely* be expreſſed in a Decimal Fraction, unleſs the Denominator of the Vulgar Fraction is either ſome Power of 2, ſome Power of 5, or elſe ſome Power of 2 into ſome Power of 5; that is, univerſally, except the Denominator is $2^m \times 5^n$; ſuppoſing *m* and *n* to denote any whole Numbers whatever, or one of them a whole Number and the other $= 0$.

For, it is very evident that, the Number 10 is diviſible by none of the Digits except 2 and 5; and it is proved *(Eu. 8. B. 14 & 15 Prop.)* if one Number meaſure another, that the Square (or Cube) of that Number will meaſure the Square (or Cube) of the other Number; from whence it follows, that any Power of the greater is diviſible by the ſame Power of the leſſer Number; and it is well known, that, in reducing any Vulgar Fraction into a Decimal Fraction, the Numerator of the former is ever multiplied (or ſuppoſed to be multiplied) by ſome Power of 10, and the Product divided by the

Hence it appears that $\frac{3}{4}$ (Three-fourths) of an Unit, are equivalent to .75 ($\frac{75}{100}$), that is, to 75 Hundredth Parts of an Unit or Integer: Alſo, $\frac{7}{8}$ (Seven-eights) of an Unit, are equal to .875 ($\frac{875}{1000}$) 875 Thouſandth Parts of an Unit.

After the very ſame Manner, may any other Vulgar Fraction be reduced into a Decimal Fraction.

ADDITION *of* DECIMALS.

PROP. 2.

To find the Sum of any given Number of Decimal Fractions, or mixed Numbers.

The Method of Operation is the very ſame as in whole Numbers, ſtrict Regard being taken in placing the ſeparating Points one under another; and alſo to place Units under Units, *&c.* in the Integers, or whole Numbers, and Tenths under Tenths, *&c.* in the Decimal Parts; and, laſtly, to point off as many Decimals in the Total, as there are in that given Term, which conſiſteth of the greateſt Number of Decimal Places.

EXAMPLE.

Suppoſe it were required to find the Sum of the following mixed Numbers and Decimal Parts, *viz.*

26.489,

---

the Denominator; hence it is plain, that unleſs the Denominator of the Vulgar Fraction be expreſſed by $2^m \times 5^n$ (*m* and *n* being any whole Numbers, *&c.*) the Numerator of the Fraction, (whatever it be) drawn into ſome Power of 10, cannot be diviſible by the ſaid Denominator; conſequently the Quotient will never terminate, and therefore muſt neceſſarily be either a repeating, or a circulating Decimal.

Hence it is evident, that in thoſe Decimals which happen to terminate, the Unit's Place of the Diviſor (or Denominator) will ever be found, either 0, 2, 4, 5, 6 or 8; but when either 1, 3, 7 or 9 ſtands in the Unit's Place of the Diviſor (or Denominator) it will be impoſſible for the Quotient (or Decimal Figures) to terminate. *N. B.* If any of the Digits, 2, 4, 5, 6 or 8, ſtands in the Place of Units in the Diviſor, the Quotient or Decimal Figures may, ſometimes, happen not to terminate.

26.489, 82.05, 18.407, .5632, .82 and .076.— Theſe Terms, being placed according to the preceding Directions, will ſtand thus

	26.489
	82.05
	18.407
	.5632
	.82
	.076
Total	128.4052

This being ſo very obvious, it would be quite unneceſſary to give any more Examples of this Kind.

SUBTRACTION *of* DECIMALS.

PROP. 3.

To find the Difference of any two given Decimal Fractions, or mixed Numbers.

RULE.

Let the ſame Method be obſerved in placing the two given Quantities, as in the preceding Rule, and if it ſo happens, that the upper (or greater) *Quantity* ſhould not conſiſt of as many Decimal Places, as the *lower*; the Defect muſt be ſupplied by annexing Cyphers (or ſuppoſing them annexed) to the upper Term; then ſubtract as if they were whole Numbers, and we ſhall obtain the Remainder, or Difference; obſerving to place the Decimal, or ſeparating

Point, exactly under thoſe of the two given Numbers.

EXAMPLES.

From	8.75	25.87	.76
Take	4.8476	12.384	.2847
Remainders	3.9024	13,486	.4753

MULTIPLICATION *of* DECIMALS.

PROP. 4.

To find the Product of any two given Decimal Fractions, or mixed Numbers.

RULE.

Let that *Factor* which conſiſteth of the greateſt Number of Figures, be multiplied by the *other*, in the very ſame Manner as if they were both whole Numbers, and from the Product point off, towards the right Hand, as many Decimals, as there are Decimal Places in the given Factors.

EXAMPLE I.

Let it be required to find the Product of theſe two Factors, *viz.* .764 and .28.

```
 .764
  .28
 ----
 6112
1528
 ----
.21392 the required Product.
```

Note. The Reaſon of pointing off as many Decimals in the Product as there are Decimal Places in both the given Factors, is very evident: For the above Fractions being expreſſed by $\frac{764}{1000}$ and $\frac{28}{100}$, and the Product of theſe (by the Nature of Vulgar Fractions) will be $\frac{21392}{100000}$; moreover, according to the Notation of Decimal Fractions, the *(ſuppoſed)* Denominator of each Fraction muſt conſiſt (as above) of as many Cyphers, as there are Figures (and Cyphers) in both Fractions; hence it is evident, that the *(ſuppoſed)* Denominator of the Product will conſiſt of an Unit, with as many Cyphers annexed, as there are Cyphers in both the *(ſuppoſed)* Denominators of the given Factors, or Decimal Places in both the given Fractions, which muſt, evidently, be equal to the Number of Decimals in the required Product.

EXAMPLE 2.

```
Multiply  24.8763
By         3.47
         -------
         1741341
         995052
        746289
         -------
Product  86.320761
         -------
```

When the Product does not conſiſt of as many Figures, or Places, as there are Decimal Places in both the given Factors, the Defect muſt be ſupplied, by prefixing Cyphers to the Product: As in the following

EXAMPLE.

Example.

Multiply 5.478
By .00054

21912
27390

Product .00295812

Division *of* Decimals.

Prop. 5.

Two Decimal Fractions, or mixed Numbers (or one of them a mixed Number, and the other either a Decimal Fraction or a whole Number) being given; to find the Quotient ariſing from dividing one *by the* other. The Method of Operation, here, is the very ſame as in Diviſion of whole Numbers; the only Difficulty lies in determining the true Value of the Quotient, or of pointing off the right Number of Decimal Places.—To effect which, obſerve the following general

Rule.

Point off as many Decimals in the Quotient, as the *Number* of Decimal Places in the Dividend exceeds *that* in the Diviſor.

For it is evident, from the preceding *Note*, that the *Decimal Places* in the Dividend muſt be *exactly* equal to the Number of *thoſe* in both the Diviſor and Quotient.

EXAMPLE I.

Divide .728654 by .34.

OPERATION.

```
.34).728654(2.1431
     68
     --
      48
      34
      ---
      146
      136
      ---
       105
       102
       ---
         34
         34
         --
         ..
         --
```

In the above Example, there are ſix Places of Decimals in the Dividend, and only two in the Diviſor; conſequently, by the general Rule, there muſt be four Places of Decimals in the Quotient.

It is very obvious, from the preceding Rule, that when there are juſt as many Decimal Places in the Dividend as there are in the Diviſor, the Quotient will be a whole Number, if there happens to be no Remainder after the Operation: But if there ſhould be a Remainder, let Cyphers be annexed thereto, and ſo continue the Diviſion at pleaſure, and we ſhall have as many Decimals in the Quotient as there were Cyphers annexed in the Operation.

EXAMPLE

EXAMPLE 2.

Let it be required to divide 1341.482 by 5.283, and to have three Places of Decimals in the Quotient.

It is very evident, from the general Rule, that there muſt be three Cyphers annexed to the Dividend; then the Operation will be as follows,

OPERATION.

```
5.283)1341.482000(253.924, &c.
      10566
      -----
       28488
       26415
       -----
        20732
        15849
        -----
         48830
         47547
         -----
          12830
          10566
          -----
           22640
           21132
           -----
            1508 Remainder
            ----
```

When there are not ſo many Figures in the Quotient, as there are Decimal Places in the Dividend more than in the Diviſor; ſupply the Defect with Cyphers, prefixed to the ſaid Quotient; as in the following

EXAMPLE 3.

Divide 7.856249 by 165.45.

OPERATION.

```
136.45(7.856249(.0575, &c.
        68225
        ------
        103374
         95515
        ------
          78599
          68225
          -----
          10374 Remainder.
```

PROP. 6.

To reduce Coin, Weight, Meaſure or Time, &c. *into Decimal Fractions.*

RULE.

When there are two, or more, different Denomitions given to be reduced into Decimal Fractions, whether they be Coin, Weight, &c. Firſt reduce all thoſe different Denominations into the loweſt of them, which will be the Numerator of a Vulgar Fraction, whoſe Denominator will be the given Integer, reduced into the ſame Denomination as the above-mentioned Numerator; this Vulgar Fraction being then reduced (by *Prop.* 1) into a Decimal Fraction, will be the Anſwer ſought.

EXAMPLE I.

Reduce 16*s*. 4*d*. into the Decimal of a Pound.

Firſt, 16*s.* 4*d.* is equal to 196 Pence, the Numerator, and in 20*s.* (the given Integer) are 240 Pence, the Denominator; then (by *Prop.* 1.) $\frac{196}{240}$ being reduced into a Decimal Fraction will be the Anſwer required.

OPERATION.

```
240(196.0000(.8166, &c. the Decimal Fraction
    1920                              [ſought.
    ----
     400
     240
     ----
     1600
     1440
     -----
      1600
      1440
      -----
      .160 Remainder.
      -----
```

EXAMPLE 2.

Let it be required to expreſs 3*q.* 14*lb.* 10*oz.* in a Decimal Fraction, when one Ton is ſuppoſed the Integer, or Unit: Or, which is ſame Thing, to find what Decimal Part of a Ton, is 3*q.* 14*lb.* 10*oz.*

Firſt, 3*q.* 14*lb.* 10*oz.* is equal to 1578*oz*, the Numerator;

And one Ton is equal to 35840*oz*, the Denominator:

Then (by *Prop.* 1.) reduce the Vulgar Fraction $\frac{1578}{35840}$, into a Decimal Fraction; ſee the following

OPERATION.

```
35840)1578.000000(.044029, &c. the Decimal
      143360                     [ſought.
      ------
      144400
      143360
      ------
        104000
         71680
        ------
        323200
        322560
        ------
           640 Remainder.
           ---
```

PROP. 7.

To find the Value of any given Decimal Fraction.

RULE.

Multiply the given Decimal by that Number (of the next inferior Denomination) which expreſſes the Value of the Integer, of which the given Decimal Fraction is a Part; and from the Product point off the Decimal Places, according to the Rule obſerved in Multiplication; and we ſhall then obtain the Value of the given Decimal in the ſame Denomination of the Multiplier: And ſo by proceeding in the ſame Manner, 'till we come to the loweſt Denomination of the propoſed Integer, we ſhall, at laſt, get the Value of the propoſed Decimal Fraction: The following Examples will render this Rule very plain.

EXAMPLE

Example 1.

Required the Value of .9495 of a Pound Sterling.

Operation.

.9495
20 Shillings, the Value of the Integer or [Pound.
18.9900
12 Pence, the Value of the Integer or [Shilling.
11.88
4 Farthings, the Value of the Integer or [Penny.
3.52

Hence it appears, that the Value of .9495 of a Pound Sterling, is 18 *s.* 11 *d.* $\frac{3}{4}$ and 52 Hundredths of a Farthing.

Example 2.

How many Quarters, Pounds, Ounces and Drams, are contained in .482 of an Hundred-Weight?

Operation.

OPERATION.

.482
4 Quarters in the Hundred, or Integer.

1.928
28 Pounds in the Quarter, or Integer.

7424
1856

25.984
16 Ounces in the Pound, or Integer.

5904
984

15.744
16 Drams in the Ounce, or Integer.

4464
744

11.904

Therefore it is found, that .482 of an Hundred-Weight is equal to 1*q.* 25*lb.* 15*oz.* 11*dr.* and 904 Thouſandths of a Dram.

EXAMPLE 3.

To find how many Weeks, Days, Hours, *&c.* are contained in .856 of a Month (*i. e.* four Weeks.)

OPERATION.

OPERATION.

.856
4 Weeks in the Month, or Integer.

3.424
7 Days in the Week, or Integer.

2.968
24 Hours in the Day, or Integer.

3872
1936

23.232
60 Minutes in the Hour, or Integer.

13.920
60 Seconds in the Minute, or Integer.

55.200

It is found in the preceding Operation, that .856 of a Month, is equivalent to 3 *W*. 2 *D*. 23 *H*. 13 *M*. 55 *S*. and 2 Tenths of a Second.

SECTION II.

Of the SQUARE ROOT.

TO find the Square Root of any given Number, is to find ſuch a Number (if poſſible) *which being multiplied by itſelf, the Product ſhall be equal to the given Number.* Thus, the Square Root of 4 is 2 (becauſe 2 multiplied by 2 is equal to 4) and for the ſame Reaſon, the Square Root of 9 is 3, and of 16 is 4, *&c.* all which will evidently appear from the following Table of Roots and Squares.

Roots	1	2	3	4	5	6	7	8	9	&c.
Squares	1	4	9	16	25	36	49	64	81	&c.

The Square Roots of Numbers are either ſimple or compound; *viz. ſimple*, when the Root conſiſts of one Figure only; and *compound*, when it contains more than one Figure: And it may be proper to obſerve here, that the Number of Places in the Square of any given Number, whether a ſimple or compound Root, will either conſiſt of juſt double the Number of Places in the ſaid Root, or one Place leſs than the ſaid double; that is, if there are two Places; the Square thereof cannot conſiſt of more than four Places, nor leſs than three; if there are three Places, the Square thereof will either conſiſt of five or ſix Figures, or Places, *&c.*——Hence it appears (ſee the ſubſequent *Lemma* *) that if a *Point* be placed over the Unit's Place of any whole Number, whoſe Square Root is to be extracted, and *another* over the third Figure, and ſo on, over the *fifth*, *ſeventh*, *ninth*, &c. *viz.* over every other Figure to the End; we ſhall have as many integral Figures (or Places) in the Root, as there were Points placed over the propoſed whole Number.

Any whole Number being thus pointed into Periods, its Square Root may be obtained by the following

GENERAL RULE.

First find by the preceding Table, or a few Trials, which of the nine Digits being ſquared, will be equal, or the neareſt *leſs*, to the firſt Period, beginning at the left Hand; which being found, place it at the right Hand of the given Number, whoſe Square Root you are then ſeeking, in the ſame Manner as a Quotient in common Diviſion. Then let the Square of this Number (which is the firſt Figure of the required Root) be taken from the firſt Period, and to the Remainder (*if any*) join the next Period to the Right-Hand, this Number is called a *Reſolvend*; double the Figure of the Root, and place it as a Diviſor to the Reſolvend; then ſeek, as in Diviſion, how often this Diviſor is contained in the Reſolvend, all but the Unit's Place, and with this Reſtriction too, that when the Quotient Figure (or this laſt Figure of the Root) is annexed to the aforeſaid Diviſor, and the Whole multiplied by the ſaid annexed Figure, the Product ſhall not exceed the Reſolvend; but ſhall either be equal thereto, or the next leſs, this Product being taken from the Reſolvend, to the Remainder, let another Period be annexed, which will then form a ſecond Reſolvend.

Double the two Figures of the Root, which place (as before) for a Diviſor to this ſecond Reſolvend; find how often this Diviſor is contained in the ſaid Reſolvend, neglecting the Unit's Place, ſtill obſerving, that when the Quotient Figure (which is the third Figure of the Root) is annexed to this laſt Diviſor, and the Whole multiplied by the Figure ſo annexed, the Product muſt be equal, or the next leſs, to the Reſolvend.

By proceeding in this Manner, Period after Period, 'till they are all brought down; and if there be no Remainder after the Operation, the Number propoſed is a ſquare Number.

If there ſhould *ſtill* be a Remainder, then the propoſed Number is called a *ſurd Number*, and has no *true* Root; but any Degree of Exactneſs may be obtained, by annexing two Cyphers to each Remainder, and proceeding as above.

EXAMPLE I.

Let it be required to extract the Square Root of 134689: Or, which is the ſame Thing, to find a Number (if poſſible) which being multiplied by itſelf, the Product ſhall be equal to 134689.

OPERATION.

The given Number being pointed in the Manner as before taught, will ſtand thus, 13̇46̇89̇, which ſhews there will be three Figures in the Root, if it happens to be a perfect Square Number; and if a ſurd Number, there will however be three Figures in the integral Part of the Root.

Here then being three Periods, *viz.* 13, 46 and 89 (or more properly 130000, 4600 and 89): Firſt find in the Table of ſimple Roots, or otherwiſe, what Number being ſquared, will be equal, or the next leſs, to the firſt Period 13, which is readily found to be 3, the firſt Figure of the Root; the Square of which (9) being taken from 13, leaves 4; to this Remainder join the next Period, and it makes 446, which is called the *Reſolvend*; and the Work will ſtand as under.

13̇46̇89̇(3
9
―――
446 Reſolvend.

Then place (6) the Double of the Root, as a Diviſor to this Reſolvend 446; and ſeek how often 6 in 44, but in ſuch a Manner that the Quotient Figure (which will be the ſecond Figure of the Root) being annexed to the Diviſor (6) and the Whole multiplied by the Figure ſo annexed, the Product muſt be either equal, or the next leſs, to the Reſolvend 446: Now, in the preſent Example, the ſecond Figure of the Root is found to be 6, and therefore the Diviſor is 66, which being multiplied by 6, and the Product (396) taken from the Reſolvend (446 leaves 50; to which join the next Period (89) and it makes 5089, for a ſecond Reſolvend; and the Operation will ſtand as follows.

$$
\begin{array}{rl}
 & \dot{1}3\dot{4}6\dot{8}9(36 \\
 & \underline{9} \\
66) & 446 \\
 & \underline{396} \\
 & 5089 \ \textit{Reſolvend.}
\end{array}
$$

Double the Figures (36) of the Root, which place as a Diviſor to this laſt Reſolvend; then find how often 72 (the Double of 36) is contained in 508, and with the ſame Reſtriction as before; *namely*, when the new Quotient Figure (which will now be the third Figure of the Root) is annexed to (72) the Diviſor, and the Whole multiplied by the Figure ſo annexed, the Product ſhall not exceed the Reſolvend, but ſhall be either equal thereto, or the next leſs: Now here it is eaſy to perceive that 7 is the next Figure of the Root (for 7 times 72 is 504); therefore annex 7 to the Diviſor, and multiply the Whole (727) by 7, the Product will be 5089, which being equal to the Reſolvend, (and all the Periods brought down) ſhews the propoſed Number (134689) is a perfect ſquare Number: See the whole Operation.

```
  .  .  .
134689(367 the required Root.
 9
 ——
66)446
   396
   ———
727)5089
    5089
    ————
    ....
    ————
```

When the given Number to be extracted, is either a mixed Number, or a Decimal Fraction, the Method of Operation will be the very ſame as in the foregoing Example; only obſerving, that if the Decimals conſiſt of an odd Number of Places, they muſt firſt be made an even Number, by annexing 1, 3, 5 or 7, *&c.* Cyphers according to the Exactneſs required in the Root; which will always conſiſt of as many Integral, and as many Decimal Places, as there were Points reſpectively placed over the Integers, and Decimal Places (together with the Cyphers annexed) of the propoſed Number. Ex-

---

LEMMA.

* *If any Number whatever be denoted by F Places; then will the Square of that Number ever conſiſt of either 2F, or 2F + 1 Places.*

It is ſufficiently evident, that if any Number of Figures, denoted by F, be multiplied by any one of the nine Digits; the Number of Places in the Product cannot be leſs than F (the Number of Figures to be multiplied) nor greater than F + 1; the Number of Places which would ariſe by multiplying F Number of Places by 10: And, it is equally plain, if the Multiplier conſiſts

Example 2.

What is the Square Root of 184.2 ?

Firſt, let three Cyphers be annexed and pointed as before directed, and the Operation will be as follows.

```
       184.2000(13.57, &c.
       1
       --
   23)84
      69
      ----
  265)1520
      1325
      ------
 2707)19500
      18949
      ------
        551 Remainder.
```

Here

ſiſts of two Places, the Number of Places in the Product cannot be more than F + 2 ; *viz.* the Number of Places which would be produced, by multiplying F Places by 100 ; nor leſs than F + 1, the Number of Places which would ariſe from F Places being multiplied by 10 ; which is the leaſt Multiplier for any two *integral* Figures.

By the very ſame Method of Reaſoning it appears, that, if the Multiplier conſiſts of three Places, the Number of Places in the Product cannot be greater than F + 3 ; the Number of Places (Figures and Cyphers) produced, by multiplying F Places by 1000 ; nor can it be leſs than F + 2, the Number of Places which would ariſe by multiply F Places by 100 : Hence the Number of Places, in the three aforeſaid Caſes, may either be F or F + 1, F + 1 or F + 2, F + 2 or F + 3, according to the Largeneſs, or Smallneſs of the laſt Figure, on the Left-Hand, in the Multiplicand and the Multiplier.

Now the greater Factor (or Multiplicand) being here denoted by F, let the leſſer Factor (or Multiplier) be called f ; then it is evident, from the preceding Method of Reaſoning, that the Number of Places in the Product will either be $F+f$, or $F+f-1$; conſequently when the Number of Places in each Factor is equal ; that is, when $F=f$ (which is the Caſe when any Number is to be ſquared) ; then the Number of Places in the Product, or in the

Here muſt be two Decimals pointed off in the Root (becauſe there are two Points over the Decimal Places) and alſo two integral Numbers, agreeable to the foregoing Obſervation.

By annexing more Cyphers, and continuing the Operation, we may approximate the Value of the Square Root of 184.2, to any aſſigned Degree of Exactneſs.

EXAMPLE 3.

To extract the Square Root of .84567.

OPERATION.

$.8\dot{4}5\dot{6}7\dot{0}$(.919 the required Root, *nearly*.

$$\begin{array}{r l}
 & 81 \\ \hline
181) & 356 \\
 & 181 \\ \hline
1829) & 17570 \\
 & 16461 \\ \hline
 & 1109 \\ \hline
\end{array}$$

Ex-

---

the Square of F *(f)* Number of Places, will either be 2F, or 2F — 1; that is, if F (or *f*) = 1 (one Figure); the Number of Places in F^2 *(f^2)* will either be 1 or 2 (*viz.* 2F — 1 or 2F); if F = 2 (two Places) the Number of Places in F^2, will either be 3 or 4 (F — 1, or 2F); and if F = 3 (three Places); then will the Number of Places in F^2 be either 5 or 6 (2F — 1, or 2F) *&c.* Q. E. I.

Corollary I.

Hence it appears, that if any ſquare Number conſiſt of either 1 or 2 Figures, or Places, its Square Root will conſiſt of one Figure *only*; if there be either 3 or 4 Places in any ſquare Number, its Square Root will have *preciſely* two Places; if either Five or Six, its Square Root will have three Places &c.

EXAMPLE 4.

What is the Square Root of 2?

OPERATION.

```
·  ·  ·  ·  ·
2.00000000(1.4142, the Square Root
1                      [of 2, nearly.
—
24(100
   96
  ———
 281(400
     281
    ———
 2824(11900
      11296
     ———————
 28282)60400
       56564
     ———————
        3836
        ————
```

---

its Square Root will have three Places, &c.—From whence appears the Reaſon of pointing the 1ſt, 3d, 5th and 7th Place, &c. beginning at the Unit's Place of any Number, whoſe Square Root is to be extracted.

COROLLARY 2.

It appears likewiſe, from the foregoing *Lemma*, if any Number of Figures be repreſented by F; that the Number of Places in F^3 cannot exceed $3F$, nor be leſs than $3F - 2$; therefore the Number of Figures in the Cube of one ſingle Figure, either will be, 1, 2 or 3 ($3F - 2$, $3F - 1$ or $3F$); the Number of Places, in the Cube of any two Figures, will be either, 4, 5 or 6 ($3F - 2$, $3F - 1$ or $3F$); and the Cube of three Figures, will conſiſt of either 7, 8 or 9 Places (*viz.* $3F - 2$, $3F - 1$ or $3F$:)—Hence the Reaſon is plain for pointing the 1ſt, 4th, 7th, 10th Place, &c. beginning at the Unit's Place of any whole Number, whoſe Cube Root is to be extracted.

SECTION III.

Of the CUBE ROOT.

TO extract the Cube Root of any given Number, is the ſame Thing as to find (if poſſible) *a Number which being multiplied by itſelf, and the Product thereof multiplied again by the ſaid Number; the laſt Product ſhall be equal to the Number given.*—— For Inſtance, the Cube Root of 27 is evidently 3; becauſe 3 multiplied by 3, and the Product (9) multiplied again by 3 gives 27: Alſo the Cube Root of 64 is 4, of 125 is 5, *&c.* ſee the following Table.

Roots	1	2	3	4	5	6	7	8	9	&c.
Cubes	1	8	27	64	125	216	343	512	729	&c.

The *Cube Roots* of Numbers are ſimple, when they conſiſt of one Figure only; and compound, when *they* contain more than one: The firſt of theſe are eaſily learnt by Heart, from the preceding Table; but the latter requires a tedious Operation: To effect which, obſerve the following Directions.

Make a Point over the Unit's Place of the propoſed Number, and another over the 4th, and ſo on, over 7th, 10th Figure, *&c.* and we ſhall have as many Integral Figures in the Root, as there are Points placed over the given whole Number.

Find a Number, which, being cubed, ſhall be equal, or the next leſs, to the firſt Period beginning at the left Hand, this Number is the firſt Figure of the Root.

The Cube of this firſt Figure of the Root being taken from the firſt Period, and to the Remainder (if any) bring down the next Period, which will then form what is called the *Reſolvend.*

Triple the Root, and alſo triple its Square, which being put down, ſo that the Unit's Place of the latter may ſtand under the Place of Tens of the former; the Sum of theſe Numbers is called a *Diviſor*, by which the next Figure of the Root may be nearly eſtimated, as follows.

Seek how often the Diviſor is contained in the Reſolvend, excluſive of the Unit's Place thereof, and with the following Reſtriction too; namely, that if the Cube of the Quotient Figure (which muſt be the ſecond Figure of the Root) be placed under the Reſolvend, Units under Units, and the Square of the Quotient Figure multiplied by the triple of the other Figure of the Root, and the Unit's Place of the Product, ſet under the Place of Tens of the aforeſaid Cube, and alſo this laſt Quotient Figure (or ſecond Figure of the Root) multiplied by the triple Square of the firſt Figure of the Root (found as above) and the Unit's Place of this Product ſet under the Place of Tens of the laſt Product; the Sum of theſe three Numbers (which is called the *Subtrahend)* muſt be equal, or the next leſs, to the Reſolvend; from which let the Subtrahend be taken, and to the Remainder (if any) bring down the next Period to form another Reſolvend; and proceed in the very ſame Manner, as above, to find the Diviſor and the third Figure of the Root, and ſo on, Period after Period, 'till they are all brought down; and then, if there happens to be no Remainder, the Number propoſed was a perfect cube Number:——But if the whole Number to be extracted, be not a perfect cube Number, three Cyphers muſt be annexed to the laſt Remainder for a new Reſolvend, and ſo proceed as above; then as many Times as there are three Decimal Cyphers annexed to the Remainders, ſo many Decimal Places will be in the Root.

EXAMPLE.

To extract the Cube Root of 3375.

OPERATION.

3375(15 the required Root.
1

2375 Reſolvend.

3 Triple of the Root 1.
3 Triple Square of the Root.

33 Diviſor.

125 The Cube of 5.
75 The Square of 5, by the triple Root.
15 Triple Square of the Root by 5.

2375 Subtrahend, to be taken from the Reſolvend.
0 Remains.

Otherwiſe, more generally; *from whence will appear the Reaſon of placing the Numbers to form the Diviſor and Subtrahend, as above.*

It is plain, if the given Number (3375) be pointed according to the foregoing Directions, there will be two Periods, *viz.* 3000 and 375, which ſhew there will be two Figures in the Root.

The next leſs Cube Number to 3000 is 1000, whereof the Cube Root is 10; therefore 1000 (the Cube of the Root 10) being taken from the firſt Period, there remains 2000; to which add the next Period (375) and we get 2375 for a Reſolvend: See the following Operation at large.

$\dot{3}37\dot{5}$(15 the required Root.
1000

2375 Reſolvend.

30 Triple of the Root 10.
300 Triple Square of the Root 10.

330 Diviſor.

125 The Cube of 5.
750 The Square of 5, by the triple Root 10.
1500 Triple Square of the Root (10) by 5.

2375 Subtrahend, to be taken from the Reſolvend.

0

Example.

To extract the Cube Root of 22425768.

Ope-

OPERATION.

22425768(282
8

14425 Reſolvend.

6 Triple of the Root.
12 Triple Square of the Root.

126 Diviſor.

512 Cube of 8.
384 Square of 8, by the triple Root.
96 Triple Square of the Root by 8.

13952 Subtrahend, to be taken from the above [Reſolvend.

473768 Reſolvend.

84 Triple of the Root.
2352 Triple Square of the Root.

23604 Diviſor.

8 Cube of 2.
336 Square of 2, by the triple Root.
4704 Triple Square of the Root by 2.

473768 Subtrahend.

0 Remains.

SECTION IV.

The CONSTRUCTION and USE of the SLIDING-RULE.

TO treat of this valuable Inſtrument from its Origin, it would be abſolutely neceſſary to explain the Nature and Properties, and to compute a Table of Logarithms; from whence the principal Lines thereon were conſtructed: But, as theſe Things might be deemed foreign to the preſent Subject, I ſhall therefore content myſelf with giving the Method of conſtructing the Lines, and afterwards of applying them to Practice: However, for the Sake of the inquiſitive Reader, I ſhall ſhew, in the *Notes* ſubjoined, the Conformity of the Operations on the Rule with the Nature of Logarithms.

The Lines on this Inſtrument, marked A, B, N, C, and two marked D, are Lines of Numbers, commonly called *Gunter's Lines*, from their worthy Inventor Mr. *Edmund Gunter*, the third Profeſſor of Aſtronomy in *Greſham College*, LONDON; who, in the Year 1624, firſt made the Diſcovery of applying Logarithms to Extenſion; and of performing, with great Facility, by Means of a Pair of Compaſſes, and the ſaid Line of Numbers, the Buſineſs of Multiplication, Diviſion, and all Arithmetical Operations, where the Rule of Proportion was required: But, the Uſe of Compaſſes being found both troubleſome and liable to Error, the late ingenious *Tho. Everard*, Eſq. made a very conſiderable Improvement in the Application of the Line of Numbers, by contriving one Line to ſlide by another,

ther, in the ſame Manner as the Inſtrument we are now ſpeaking of.

The Method of conſtructing the Line of Numbers is the very ſame, let the Radius, or Length of the Line, be what it will; thoſe *Lines* on the Sliding-Rule, marked A, B, N, and C, are graduated upon Half the Radius as *thoſe* marked D.

Let a Line, or Rule (equal to the whole Diſtance or the intended Radius), upon which the Line of Numbers is to be graduated, be divided into 1000 equal Parts; then with your Compaſſes take, from this Line of equal Parts, the Numbers expreſſing the Logarithms (to the firſt three Places of Decimals, omitting the Characteriſtics) of 101, 102, 103, 104, *&c.* progreſſively to 1000, and apply them ſucceſſively from 1 (the Beginning of the Radius), and we ſhall thereby mark out all the Diviſions on the ſingle Radius D: But the moſt expeditious and exact Method of forming a Line of Numbers, is as follows.

Open the Sector 'till the Diſtance of the two Braſs Pins, on the Line of Lines (marked L. L.) be equal to the Length of the intended Radius; place 1 (the Logarithm of which is 0) at the Beginning of the Line, towards the left Hand; then, according as the Space between 1 and 2 is divided into 100 or 50 Parts (as in the ſingle Radius marked D, or thoſe Radii marked A, B, *&c.*), take from the Sector, opened to the intended Radius, the Diſtances, or Numbers, anſwering to (at leaſt the three firſt Places of Decimals) the Logarithms of 1.01, 1.02, 1.03, 1.04, *&c.* to 2 (if the ſingle Radius D); or thoſe of 1.02, 1.04, 1.06, 1.08, *&c.* to 2 (if any of the Radii marked A, B, *&c.*); then theſe Diſtances being ſucceſſively applied from 1, along the Line D or A, will mark out all the Diviſions between 1 and 2 on thoſe Lines reſpectively.

Now the Diſtance of the Diviſions 2 and 3, in the ſingle Radius (marked D), is divided into 50 Parts; but in thoſe Radii marked A, B, &c. the ſaid Diſtance is divided into 20 Parts; therefore, for the former, take from the Sector, opened for the ſingle Radius, the Logarithms of 2.02, 2.04, 2.06, 2.08, &c. to 3; and for the latter, take from the Sector, opened for the double Radius, the Logarithms of 2.05, 2.1, 2.15, 2.2, 2.25, &c. to 3, and apply theſe Diſtances reſpectively from 1, along the Lines D and A, and we ſhall thereby obtain all the Diviſions between 2 and 3: Moreover, the Diſtance between 3 and 4, in each Radius on the Rule, is divided into 20 Parts; therefore take from the Sector (opened to its proper Radius) the Logarithms of the Numbers 3.05, 3.1, 3.15, 3.2, 3.25, &c. to 4, and apply them (as before) from 1, along either of the Lines D, A, or B, and they will point out all the Diviſions between 3 and 4: By proceeding in this Manner, the Diviſions between 4, 5; 5, 6; 6, 7; 7, 8; 8, 9; and 9 and 10, may be eaſily marked out.

The Line marked E, conſiſting of three equal Radii, is conſtructed in the very ſame Manner as the above: For the Sector being opened to one-third of the Extent of the ſingle Radius D; take off the Logarithms of 1.05, 1.1, 1.15, &c. to 2, and apply thoſe Diſtances from 1, 10, or 100, in each Radius, and they will mark out all the Diviſions between 1 and 2, 10 and 2, and 100 and 2: — Again, the Diſtance between 2 and 3 is divided into 20 Parts; proceed therefore according to the foregoing Directions, and we ſhall obtain all the Diviſions between thoſe two Numbers;

and

and in like Manner between 3, 4; 4, 5; and 5 and 6, *&c.*

On the Line marked M.D, are alſo placed Lines of Numbers, only they ſtand in an inverted Order, beginning at 21.5042; the laſt Diviſion on the right Hand will then repreſent .215042: The Uſe of this Line (MD), together with A and B (or N), is wholly confined to the Gauging of Malt, in Veſſels in the Form of rectangular Parallelopipedons, at one Setting of the Rule; but the ſame Examples may be performed, with more Eaſe to a Learner, by the Lines A and B only; or with the Lines D and C, after finding a geometrical mean Proportional between the Length and Breadth of the Baſe.

The Lines of Segments, marked S.S. and S.L. (ſignifying *Segment Standing* and *Segment Lying,*) are uſed in finding the Ullage of a Caſk, or the Quantity of Liquor which a Caſk wants of being full, or what Quantity is in it, if not quite full.

Theſe Lines of Segments may be laid down in the following Manner. Take a Caſk whoſe Content is 100 Ale (or Wine) Gallons, and the neareſt in Form to thoſe which moſt frequently occur in Practice; then ſuppoſe the Bung Diameter or Length (according to the Poſition of the Caſk) divided into 100 equal Parts, which muſt be laid down on the Slide marked N, in a logarithmic Manner, by the Method already preſcribed, *Page* 32.

Draw out, and carefully meaſure, ſucceſſively, the Quantities contained in the 1ſt, 2d, 3d, 4th, and 5th, *&c.* of thoſe equal Diviſions of the Bung Diameter (or Length); then place the Quantity contained in the 1ſt, in the 2 firſt, 3 firſt, 4 firſt, *&c.* of thoſe equal Diviſions *exactly* againſt the Numbers 1, 2, 3, 4, *&c.* reſpectively, on the

Slide N; and ſo, by proceeding in this Manner, we ſhall obtain the true Quantity in ſuch a Caſk to every hundredth Part of its Bung-Diameter, or Length. — And if either of theſe were ſuppoſed to be divided into any other Number of equal Parts (beſides 100), and the Quantities contained in the firſt, 2 firſt, 3 firſt, &c. of thoſe equal Parts, be placed exactly oppoſite the Numbers, 1, 2, 3, &c. reſpectively, on the Slide N; we ſhould thereby obtain a Table of Segments for a ſtanding (or lying) Caſk ſimilar to the former.

Whence the Reaſon of finding what is called the *Segment*, is very evident: For it is only conceiving the Bung Diameter (or Length) of that Caſk, from which the Lines of Segments were ſuppoſed to be conſtructed, to be divided into as many equal Parts as there are Inches, &c. in the Bung Diameter (or Length) of the Caſk, whoſe Ullage we are then ſeeking, and placing that Number againſt 100 on the Segments; then oppoſite any propoſed Number of wet Inches, &c. (or equal Parts of the Bung Diameter or Length) we ſhall have the Segment ſought.

On various Parts of the Rule are ſeveral remarkable Points; ſome of which are diſtinguiſhed with Braſs Pins and Letters, others with only ſmall Dots and Letters.

Thus, on the Line A there is marked MB, with a Braſs Pin, at 2150.42, the cubic Inches in a Malt Buſhel; alſo on the ſame Line is fixed a Braſs Pin, with the Letter A at 282, the cubic Inches in the Ale Gallon.

On the Line B is a ſmall Dot marked at .707, and alſo the Letters *S. i.* which ſignify *Square inſcribed*; uſeful in finding the Side of a Square inſcribed in any given Circle: At .886 is a ſmall Dot, and likewiſe the Letters *S. e.* which denote

Square equal; uſeful in finding the Side of a Square, whoſe Area ſhall be equal to that of any given Circle: At 3.1416 is a ſmall Dot marked C, ſignifying *Circumference*, neceſſary for finding the Circumference of a Circle to any given Diameter.

On the Line D are placed ſeveral Gauge-points, diſtinguiſhed by Braſs Pins and Letters: *Viz.* W.G. with a Braſs Pin, is placed at 17.15, being the Wine Gauge-point for Circles; and A.G. marked at 18.95, ſignifying the Ale Gauge-point for Circles, *&c.* At 46.37 are the Letters M.S. which ſignify *Malt Square*, being the Malt Gauge-point for Square Meaſure; At 52.32 ſtand M.R. which denote *Malt Round*, being the Malt Gauge-point for circular Meaſure; Alſo, at 6.32 ſtand T.P. which ſignify *Tallow Pounds*, being the neat Tallow Gauge-point for circular Figures.

On the Slide C there is a ſmall Dot, with the Letters O.C. marked at .07957, which is the Area of a Circle whoſe Circumference is Unity; uſeful in finding the Area in Inches, Feet, *&c.* of any Circle whoſe Circumference is known: On the ſame Line is marked O.*d.* at .7854, the Area of a Circle whoſe Diameter is Unity; this is uſeful in finding the Area in Inches, Feet, *&c.* of any Circle whoſe Diameter is given.

The Method of eſtimating the Values of the Diviſions on the Sliding-Rule; and the Uſe thereof.

WHatever Value is aſſigned to the firſt 1, towards the left Hand, (whether 1, 10, 100, *&c.*) on the Lines marked A, B, N, *&c.* the following integral Numbers, 2, 3, 4, *&c.* will repreſent twice, thrice, four times, *&c.* as much; and conſequently the ſecond 1 (if a double Radius) will be 10 times the Value of the firſt; and the third 1 (if a triple Radius) will be 100 times the Value of the firſt, or 10 times the Value of the ſecond 1. The Values of the integral Diviſions being thus eſtimated, thoſe of the intermediate Diviſions may be eaſily known; being always the Quotient expreſſed by the Value of the Difference of two adjoining integral Numbers, divided by the Number of Parts contained between them.

Thus, for Example, if the firſt 1, at the End of the Line A (B or N), ſtands for *one*, the following 2 for *two*, *&c.* then the Number of Diviſions between 1 and 2 being 50, and the Value of the Difference of thoſe integral Diviſions is 1; therefore the Value of one of the intermediate Diviſions is $\frac{1}{50}$th; conſequently the Values of the 1ſt, 2d, 3d, 4th, 5th, *&c.* Diviſions from 1, will be expreſſed by $1\frac{1}{50}$, $1\frac{2}{50}$, $1\frac{3}{50}$, $1\frac{4}{50}$, $1\frac{5}{50}$, *&c.*

Again, the Space between the ſecond 1 and 2 (which Numbers, according to the laſt Eſtimation, repreſent 10 and 20) is alſo divided into 50 Parts; that is, firſt into 10 large Diviſions, and then each of thoſe into 5 Parts; then the Difference of the integral Numbers (10 and 20) being 10; therefore the Value of *one* large Diviſion will be 1 or $\frac{10}{10}$, (*i. e.* 10 divided by 10), and the Value of one ſmall Diviſion is $\frac{10}{50}$ or $\frac{1}{5}$; conſequently the Values of the 1ſt, 2d, 3d, 4th, 5th, *&c.* Diviſions from 10, will (in this Caſe) be expreſſed by $10\frac{1}{5}$, $10\frac{2}{5}$, $10\frac{3}{5}$, $10\frac{4}{5}$, 11, *&c.* Moreover, if the ſaid integral Numbers 1 and 2, denote 100 and 200; then the Value of one of the (ten) large Diviſions will be expreſſed by 10, or $\frac{100}{10}$; and the Value of one of the (fifty) ſmall Diviſions will be expreſſed by 2, or $\frac{100}{50}$, therefore the Values of the 1ſt, 2d, 3d, 4

4th, *&c.* Diviſions from 100, will be repreſented by 102, 104, 106, 108, *&c.* — By the very ſame Method of proceeding, the Values of the intermediate Diviſions, between any two adjoining integral Numbers, may be known.

Multiplication by the Lines A *and* B, *on the Sliding-Rule.*

Prop. I.

To find the Product of two given Numbers, by the Sliding-Rule.

Rule.

To either of the given Numbers (or Factors) on A, ſet 1 on B; then againſt the other on B, is the required Product on A.

Example 1.

Required the Product of 3 by 8, by the Sliding-Rule.

Set 1 on B, to 3 (or 8) on the Line A; then againſt 8 (or 3) on B, is 24 on A.*

It may be proper to obſerve, that it will frequently happen, when 1 on B is ſet to either of the given Factors on A, the other cannot (according to the *true* Numeration of the Rule) be expreſſed on the Line B; or, being ſound thereon, it may perhaps fall beyond the Line A; in ſuch Circumſtances it will be moſt convenient, after ſetting Unity on B to one of the given Factors on A, to divide the other by ſome Power of 10[†] (*viz.* 10, 100, 1000, *&c.*) 'till the Quotient can be found oppoſite ſome Diviſion (or Product) on A; then that Product, thus ariſing, muſt be multiplied by the very ſame Power of 10 as the given Factor was divided by. One Example will make this Obſervation ſufficiently plain.

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* By drawing out the Slide, till 1 on B is oppoſite to 3 on A; it is evident we thence obtain the Sum of the Diſtances 1 to 3 on A, and 1 to 8 on B: But theſe Diſtances are reſpectively as the Logarithms of 3 and 8; and it is well known that the Sum of the Logarithms of two Numbers will expreſs the Logarithm of their Product; $\therefore$ the Sum of the Diſtances, 1 to 3 on A, and 1 to 8 on B, will be as the Log. $3 +$ Log. 8 ($=$ Log. 3×8) $=$ Log. 24.

Example 2.

To find, by the Sliding-Rule, the Product of 120 by 95.

Firſt, if 1 on B is ſet to 120 on A, then will the other Factor (95) fall beyond the Line A:—Again, if 1 on B is ſet to 95 on A, then the other Factor (120) cannot be found on B; becauſe the greateſt Number (in a double Radius) cannot exceed 100, when the firſt Radius begins with Unity.

But by ſetting 1 on B to either of the given Factors on A; then, againſt the other Factor divided by 10, (which in this Caſe is ſufficient,) we ſhall have $\frac{1}{10}$th Part of the Product ſought. Thus, ſet 1 on B to 120 on A, and againſt 9.5 on B, is 1140 on A, which, being multiplied by 10, gives 11400, the required Product.

[†] If the given Factors be called m and n, the Product of them r, and the Index of any Power of 10 be denoted by a: Then we ſhall have, $1 : m :: n : r$ (or $1 : n :: m : r$); $\therefore 1 : m :: \frac{n}{\overline{10}|^{a}} : \frac{r}{\overline{10}|^{a}}$, (or $1 : n :: \frac{m}{\overline{10}|^{a}} : \frac{r}{\overline{10}|^{a}}$); hence $\frac{r}{\overline{10}|^{a}} = \frac{m \times n}{\overline{10}|^{a}}$; conſequently, $\frac{r}{\overline{10}|^{a}} \times \overline{10}|^{a}$ $\left(= \frac{m \times n \times \overline{10}|^{a}}{\overline{10}|^{a}} = m \times n\right) = r$. Q. E. I.

It may be proper to obſerve, that whether one of the given Numbers be ſet to Unity on the Line A or the Line B, the other given Number (or Factor) muſt be found on the ſame Line where 1 (or Unity) was taken.

Diviſion by the Lines A *and* B, *on the Sliding-Rule.*

PROP. II.

To find the Quotient of two given Numbers, by the Sliding-Rule.

RULE.

To the Diviſor on A ſet 1 on B; then againſt the Dividend on A is the Quotient on B.

EXAMPLE I.

Let the Dividend be 75, and the Diviſor 5; required the Quotient.

Set 1 on B, to 5 on A; and againſt 75 on A, is 15 on the Line B, the Quotient ſought.[‡]

It will ſometimes happen, that when 1 on B is ſet to the Diviſor on A, the Dividend cannot (according to the *true* Numeration of the Rule) be found on the Line A; therefore, in this Caſe ‖, it will be neceſſary to divide the given Dividend by ſuch a Power of 10, as will bring the Quotient thereof upon the Line A; then againſt this Quotient (*viz.* of the Dividend, divided by 10, 100, 1000, *&c.*) is a Number on B, which being multiplied by the ſame Power of 10 as the given Dividend was divided by, we ſhall then obtain the true Quotient ſought.

[‡] When the Slide is drawn out till 1 on B is oppoſite 5 (the Diviſor) on A, we ſhall then get the Difference of the Diſtances of 1 to 5 on A, and alſo 1 to 75 on the Line A, but expreſſed on the Line B from 1 to 15. Now theſe Diſtances are reſpectively as the Logarithms of 5, 75, and 15, and the Difference of the Logarithms of two Numbers is equal to the Logarithm of their Quotient; ∴ the Difference of the Diſtances of 1 to 75 and 1 to 5 on A (= Log. 75 — Log. 5) will be as the Log. of 15 on B.

EXAMPLE 2.

What is the Quotient of 385 divided by 7?

To 7 on A, ſet 1 on B; then as 385 cannot be expreſſed on A, becauſe the ſecond Radius, in this Caſe, ends with 100; therefore let that Number be divided by 10; and oppoſite 38.5 (the Quotient) on A, is 5.5 on B, which being multiplied by 10 gives 55, the Quotient ſought.

It is to be obſerved here, that (whether the Diviſor on A is ſet to 1 on B, or the Diviſor on B is ſet to 1 on A) the Quotient muſt always be found on the ſame *Line* where 1 was taken, and the Diviſor and Dividend on the *other*.

One Example in the *Rule of Three* will be ſufficient; ſince the Method of Operation by the Sliding-Rule, is very nearly the ſame as in Multiplication: The only Difference is, that inſtead of ſetting 1 on B, to one of the given Factors on A, we muſt ſet the firſt of the three given Terms on

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‖ Let m denote the Diviſor, n the Dividend, and let the Quotient thereof $= r$; alſo let the Index of any Power of 10 be denoted by a; then $m : 1 :: n : r$, or $m : 1 :: \frac{n}{10|^{a}} : \frac{r}{10|^{a}}$; $\therefore \frac{n}{10|^{a}} = \frac{mr}{10|^{a}}$, or $\frac{n}{m} = r$. Q. E. I.

B, to either the 2d or 3d given Number (or Term) on A; then againſt the other Number on B, is the 4th Number, or Anſwer ſought, on A.

EXAMPLE.

If 4 Yards of Cloth coſt 14 Shillings, what will 28 Yards coſt, at the ſame Rate?

Set 4 on B, to 14 on A; then oppoſite 28 on B, is 98 on A, the Anſwer ſought §. — Or, ſet 4 on B to 28 on A; then againſt 14 on B, is 98 on A, (or 4*l.* 18*s.*) the ſame as before.

If it ſhould ſo happen, when the firſt Term (or Number) on B, is ſet to the ſecond or third Number on A, that the other Number on B falls beyond the Stock, or the Line A; then, in ſuch Circumſtance, let that Number, which ſo falls off the Rule, be multiplied, or divided (according as it falls off towards the left or right Hand) by ſome Power of 10; and againſt the Product, or Quotient, on B, is a fourth Number on A, which being divided or multiplied by the ſame Power of 10 as the forementioned Number was multiplied or divided by; the Quotient, or Product, will be the 4th Number, or Anſwer ſought. ¶

Note.

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§ By drawing out the Slide 'till 4 on B ſtands oppoſite 14 on A, we thence obtain, on the Line A, the Diſtance from 1 to 14, *plus* the Diſtance from 1 to 28, *minus* the Diſtance of 1 and 4 (on B); but theſe Diſtances are reſpectively as the Logarithms of 14, 28, and 4; ∴ the Log. 14 + Log. 28 — Log. 4 = Log. $\frac{28 \times 14}{4}$ = Log. 98 on A.

¶ Let four proportional Numbers be repreſented by *m*, *n*, *s*, and *r*; alſo let the Index of any Power of 10 be denoted by *a*: Then we have $m : n :: s : r$, (or $m : s :: n : r$); ∵ $m : n :: \frac{s}{10|^{a}} : \frac{r}{10|^{a}}$, (or $m : s :: \frac{n}{10|^{a}} : \frac{r}{10|^{a}}$); ∴ $\frac{sn}{10|^{a}} = \frac{mr}{10|^{a}}$, or $sn = mr$.

Moreover,

Note. It makes no Difference whether the firſt Number be taken on the Line A or B; only obſerve, that the 4th Number, or Anſwer, muſt be found on the contrary Line to that, whereon the firſt Number was taken. — But, in finding the Areas of plane Figures in Ale Gallons, Malt Buſhels, *&c.* (as will be ſhewn farther on,) it will be found moſt convenient to take the firſt Number, *viz.* 282, 2150, on the Line A, as there are generally braſs Pins fixed at thoſe Numbers.

To extract the Square Root, by the Sliding-Rule.

Set 10 on C to 10 on (the Stock) D; then againſt any propoſed Number on C, is its Square Root on D.

It will be proper to obſerve here, that if the Number, whoſe Square Root is required, conſiſts of an odd Number of integral Places, its Square Root will be found oppoſite the firſt Radius on the Line C: But if there be an even Number of Places, in the Number whoſe Square Root is ſought, then will that Root fall againſt the ſecond Radius on the Line C. *

Moreover, $m : n :: s \times \overline{10}|^{a} : r \times \overline{10}|^{a}$; $\therefore n \times s \times \overline{10}|^{a} = m \times r \times \overline{10}|^{a}$; conſequently $ns = mr$. Q. E. I.

* The 1 at the End of the Line D, may denote either 1, 10, 100, 1000, *&c.* therefore the firſt 1 on C oppoſite thereto, muſt, by the Conſtruction of the Lines, ſignify either 1 (1^{2}), 100 ($\overline{10}^{2}$), 10000 ($\overline{100}^{2}$), *&c.* and conſequently the ſecond 1 on C, will repreſent either 10, 1000, 10000, *&c.*

EXAMPLE.

What is the Square Root of 15376?

Let the Rule be ſet as above directed; then it is evident the firſt 1 on C will repreſent 10000, and the 1 on D (oppoſite thereto) is its Root, which now repreſents 100; likewiſe 5000 will be repreſented by 5 of the large Diviſions on C, and 376 will very nearly be repreſented by 2 of the ſmall Diviſions; then againſt this Point on C, we have 124 on the Line D; the Root ſought.

To extract the Cube Root, by the Sliding-Rule.

Set 10 on (the Slide) D, to 1000 on E; then againſt any propoſed Number on E, is its Cube Root on D.

It will alſo be proper to obſerve here, that if the Number, whoſe Cube Root is ſought, conſiſts of either 1, 4, 7, 10, *&c.* integral Places, its Cube Root will be obtained oppoſite the firſt Radius on E; and if the Number contains either 2, 5, 8, 11, *&c.* Places, its Cube Root will be found oppoſite the ſecond Radius on E; but the Cube Root of a Number, conſiſting of either 3, 6, 9, 12, *&c.* Places, will be had oppoſite the third Radius on E. [†]

EXAMPLE.

[†] The 1 on (the Slide) D, may repreſent either 1, 10, 100, 1000, *&c.* and therefore the firſt 1 on E, will repreſent either 1 (1^3), 1000 ($\overline{10}^3$), 1000000 ($\overline{100}^3$), *&c.* therefore the ſecond Radius on E muſt begin with either 10, 10000, 10000000, *&c.* and conſequently the third Radius will begin with either 100, 100000, 100000000, *&c.* Q. E. D.

EXAMPLE.

What is the Cube Root of 3375?

Set the Line D (on the Slide) exactly even with the Line E; then, againſt 3375 on the firſt Radius on E, according to the preceding Obſervation, we have 15 on D, the required Root.

The Lines C and D are likewiſe very uſeful in finding a geometrical mean Proportion between any two given Numbers; alſo in finding, from any three given Numbers, a fourth, which ſhall be to the third, as the Square of the ſecond is to the Square of the firſt Number; and therefore theſe Lines are applicable to the finding the Areas of Circles, (which are as the Squares of their Diameters) and the Contents of ſuch Solids, whereof the Square of one Dimenſion, being multiplied into another Dimenſion, ſhall expreſs either the whole Content, (as in an upright ſquare Priſm) or ſome Multiple of it, as a *Cylinder*, *Cone*, *Sphere* and *Spheroid*; and conſequently, theſe Lines may be applied in finding the correſponding Dimenſions of ſimilar Surfaces.

The Lines D and E are neceſſary in finding, from any three given Numbers, a fourth Number, which ſhall be to the third, as the Cube of the ſecond is to the Cube of the firſt; and conſequently of determining the Contents of ſimilar Solids, which are in the direct Proportion of the Cubes of their correſponding Dimenſions; and likewiſe, on the contrary, of finding the correſponding Dimenſions of ſimilar Solids.

PROP. III.

To find a geometrical mean Proportion between two given Numbers; or, which is the ſame Thing, to find the

the Square Root of the Product of any two given Numbers.

RULE.

Set one of the given Numbers on C, to the like Number on D; then againſt the other given Number on C is the geometrical mean Proportion ſought, on D.

EXAMPLE.

What is the geometrical mean Proportion between 4 and 9?

Set 4 on C, to 4 on D; then againſt 9 on C is 6 on D, the Anſwer ſought.[‡]

PROP. IV.

To find, to any three given Numbers, a fourth Number, which ſhall be to the third, as the Square of the ſecond is to the Square of the firſt.

RULE.

To the firſt Number (or Root) on D, ſet the third on C; then againſt the ſecond Number (or Root) on D, is the fourth Number ſought on C.

EXAMPLE.

[‡] By placing 4 on C, to 4 on D, we get the Sum of the Diſtances from 1 to 2 on the Line D, and alſo from 1 to 9 on the Line C; the former being $\frac{1}{2}$ the Diſtance of 1 to 4 on D, and the latter (being on the double Radius) is the ſame as 1 to 3, meaſured on the Line D; but theſe Diſtances are reſpectively as the Logarithms of 2 and 3; therefore the Log. 2 + Log. 3 ($=$ $\frac{1}{2}$ L. 4 + $\frac{1}{2}$ L. 9) $=$ Log. $\overline{4 \times 9}|^{\frac{1}{2}}$ $=$ Log. 6.

Example 1.

Suppoſe the given Numbers were 3, 9, and 12, and that it was required to find a fourth Number, which ſhall be in the ſame Proportion to 12, as the Square of 9 is to the Square of 3; that is, as 81 to 9.

To 3 on D, ſet 12 on C; then againſt 9 on D, is 108 on C, the Anſwer ſought.

Example 2.

If 3 Feet of cylindrical dried Oak, whoſe Circumference is 32 Inches, weigh 80 lb. what will 3 Feet of the ſame Sort of Oak weigh, when the Circumference is 22 Inches?

The Altitudes of the two Cylinders being equal to each other, therefore their Solidities, and conſequently their Weights, muſt be to each other as the Areas of their Baſes, which are as the Squares of their Diameters, or Circumferences.

To 32 on D, ſet 80 lb. on C; and againſt 22 on D is 37.8 lb. on C, the Weight ſought.

If it ſhould happen that, when the third Number on C is ſet to the firſt Number on D, (according to the foregoing Rule) the ſecond Number on D falls beyond the Slide, or Line C; then, in ſuch Caſe, we need only to multiply, or divide (according as it falls off the Rule towards the left or right Hand) the ſaid ſecond Number (or Root) by ſuch a Number, that the Product (or Quotient) thereof may be found on D, oppoſite ſome Number on C, which Number being divided, or multiplied, by the Square of that Number, by which the ſecond was multiplied, or divided; the Quotient, or Product, will be the Anſwer ſought.[*]

Suppoſe, in the firſt of the two preceding Examples, the ſecond Number was 15, and the other two the ſame as before. — To 3 on D, ſet 12 on C; then againſt 7.5 (the Half of 15) on D, is 75 on C, which being multiplied by 4 (the Square of 2) gives 300, the Anſwer ſought.

It will, in many Caſes, be moſt convenient to multiply, or divide, the ſecond Number by 10, and then find (as above directed) the Number on C, oppoſite that Product, or Quotient; which Number being divided, or multiplied by 100 (the Square of 10), gives the Anſwer ſought.

The above Method renders the Buſineſs of finding what are called *new Gauge-points* quite unneceſſary, as ſhall be explained farther on.

PROP. V.

Any three Numbers being given to find a fourth, ſo that the Square thereof ſhall be to the Square of the third, as the ſecond Number is to the firſt.

RULE.

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[*] Let the three given Numbers be denoted by m, n, and r, and the Number ſought by v, and alſo let *that* by which the ſecond Number is either multiplied or divided, be repreſented by d: Then, (by the *Prop.*) $m^2 : n^2 :: r : v$; but $m^2 : \frac{n}{d} \times \frac{n}{d} :: r : \frac{v}{d^2}$; alſo $m^2 : dn \times dn :: r : dv^2$; whence it is plain, that, when the ſecond Number (n) is only $\frac{n}{d}$, the fourth Number (v) will then only be the d^2 Part of *that*, when the ſecond Number is n; and likewiſe when the ſecond Number is $= d \times n$, the fourth Number, or Anſwer ſought, will then become d^2 times *that*, when the ſecond Number is equal only n. Q. E. I.

RULE.

To the third Number (or Root) on D, ſet the firſt Number on C; then againſt the ſecond on C, is the required fourth Number (or Root) on D.

EXAMPLE I.

Let the Side of a Triangle, whoſe Area is 15 Gallons, be 40 Inches; what will be the correſponding Side of another ſimilar Triangle, the Area of which is to be 60 Gallons.

Here the three given Numbers are 15, 60, and 40: Therefore, according to the above Rule, to 40 on D ſet 15 on C, and againſt 60 on C is 80 on D, the required Side.

If, when the Rule is ſet as above, the ſecond Number on C falls off the Line D; then let the third Number be multiplied, or divided, (according as the ſecond Number on C falls off towards the left or right Hand,) by ſuch a Number, ſo that, if to the Product, or Quotient, thereof, the firſt Number on C be ſet, the ſecond Number on C may fall oppoſite ſome Number on D; which being divided, or multiplied, by that Number with which the third was multiplied, or divided; the Quotient, or Product, will be the Anſwer ſought.*

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* Let every Thing be interpreted as in the preceding *Note.* Then we have (by the *Prop.*) $m : n :: r^2 : v^2$; but $m : n :: \frac{r}{d} \times \frac{r}{d} : \frac{v}{d} \times \frac{v}{d}$; likewiſe, $m : n :: dr \times dr : dv \times dv$; $\therefore$ it is evident, that, when the third Number is only $\frac{r}{d}$ (inſtead of r,) the *fourth Number* (v) will be only the dth Part of *that* when the third Number is r: Moreover, when the third Number (r) is $= d \times r$, the *fourth Number* (v) will then be $= d$ times *that* when the third Number is only r. Q. E. I.

EXAMPLE 2.

Let the three given Numbers be 15, 90, and 60; required to find a fourth Number, ſo that the Square thereof ſhall be to the Square of 60, as 90 is to ~~60.~~ 15

To 60 on D, ſet (according to the preceding Rule) 15 on C; then 90 on C, manifeſtly, falls beyond the Line D. But, if to 15 ($\frac{1}{4}$ of 60, the third Number) on D, be ſet 15 on C; then oppoſite 90 on C, we have *36.75 (very nearly)* on the Line D; which being multiplied by 4 gives 147, the required Number, *nearly*. — For 15 is to 90, as 3600 (the Square of 60) is to 21660 (the Square of 147,) *nearly*.

PROP. VI.

Let there be any three Numbers given, to find a fourth, which ſhall be to the third, as the Cube of the ſecond is to the Cube of the firſt Number.

RULE.

Set the firſt given Number (or Root) on the Slide D, to the third Number on E; then oppoſite the ſecond Number (or Root) on D, is the fourth Number required on E.

EXAMPLE.

Suppoſe the given Numbers to be 3, 6, and 18; it is required to find a fourth Number, which ſhall be to 18 as the Cube of 6 is to the Cube of 3; or as 216 to 27.

Set 3 on the Slide D, to 18 on E; then againſt 6 on D, is 144 on E, the fourth Number ſought.*

Prop. VII.

Given any three Numbers, to find a fourth, the Cube whereof ſhall be to the Cube of the third, as the ſecond Number is to the firſt.

Rule.

Set the third Number (or Root) on the Slide D, to the firſt Number on E; then oppoſite the ſecond on E, is the fourth Number ſought on D.

Example.

Suppoſe, in the Fruſtum of a Cone, there are given the bottom Diameter 40, the top Diameter 25, and the Altitude 30 Inches; it is required to find the Dimenſions of another ſimilar Fruſtum (that is, the Diameters and Altitude to remain in the above Proportion to one another,) whoſe Content ſhall exceed the former 50 Wine Gallons.

The Content of the given Fruſtum (by the Methods given farther on) is 109.6 Wine Gallons; therefore the Content of the required ſimilar

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* It is evident, that, by ſetting 1 on D, to 18 on E, we ſhall obtain the Sum of the Diſtances 1 to 18 on E, and 1 to 6 on the Line D; which laſt is equal to 3 times the Diſtance from 1 to 6 on E; but by moving the Slide (towards the left Hand) till 3 on D is oppoſite 18 on E, we thereby diminiſh the Sum of the two ſaid Extenſions by the Diſtance of 1 to 3 on D (anſwering to 3 times the Diſtance from 1 to 3 on E), and moreover get the Diſtance from 1 to 18 on E, *plus* 3 times the Diſtance from 1 to 6 on E, *minus* 3 times the Diſtance from 1 to 3 on E: But, by the Conſtruction of the Lines, theſe Diſtances are reſpectively as the Log. of 18, 3 × Log. 6, and 3 × Log. 3; whence, by the Property of Logarithms, the Log. 18 + 3 × Log. 6 − 3 × Log. 3 $\left(= \frac{18 \times \overline{6}^3}{\overline{3}^3} = 144\right)$ = Log. 144.

Fruſtum will be 159.6 Wine Gallons: — Then, by the preceding *Propoſition*, the three given Numbers ſtand thus:

109.6, 159.6, 40, to find the bottom Diameter.
109.6, 159.6, 25, to find the top Diameter.
109.6, 159.6, 30, to find the Altitude.

Set 40 on the Slide D, to 109.6 on E; then againſt 159.6 on E is 45.3 on D, *nearly*: — Set 25 on D, to 109.6 on E; then oppoſite 159.6 on E is 28.4 on D, *nearly*; — Laſtly, ſet 30 on D, to 109.6 on E; then againſt 159.6 on E, is 34 on D, *nearly*.

Hence the required Dimenſions are 45.3, the bottom Diameter; 28.4 the top Diameter; and 34 Inches the Altitude, *nearly*.

It may be proper to take Notice, that what has been already ſaid (*Prop.* 4.) with Reſpect to the ſecond Number falling off the Line C, holds equally good with Regard to the Lines D and E; only obſerve, here, to multiply, or divide, by the Cube (inſtead of the Square) of that Number by which the ſecond was divided, or multiplied.

SECTION V.

Of Geometrical Definitions *of Lines, Angles, Surfaces, and Solids.*

Definitions.

Of Lines and Angles.

1. A *Line* is Diſtance, or Length without Breadth; the Extremes, Bounds, or Limits of which, are called *Points.*

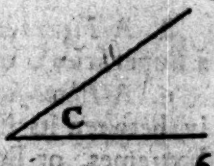
Therefore,

2. A *Mathematical Point* has no Parts.

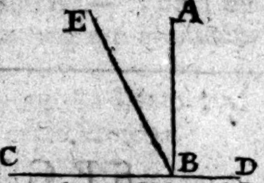
3. A *Right-line* (or *Straight-line)* is that which lies perfectly even between its Extremes, or Limits, as AB.

4. A *curved Line* is that which, in every Part thereof, lies unevenly between its Extremes, or Bounds, as *ab.*

5. A *right-lined Angle* is that which is formed by the Inclination of two Right-lines, meeting each other in a Point, as C.

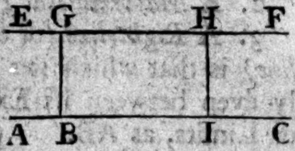


6.



6. There are three Sorts of right-lined Angles. —— 1. When one Right-line AB, ſtands any-where upon another CD, ſo as to incline no more towards one End than the other, making the Angles on both Sides AB equal, then thoſe Angles are called *Right-angles*; and the two Right-lines, AB and CD, are then ſaid to be perpendicular to each other. — 2. When the Angle (EBD) is greater than a Right-angle (ABD), it is called an *Obtuſe-angle*. — 3. If the Angle (EBC) is leſs than a Right-angle (ABC), it is called an *Acute-angle*.

Note. When an Angle is denoted by three Letters (ABC), that in the Middle ſtands at the angular Point, and the other two ſtand at the Extremities of the Lines which form the Angle: Thus, in the preceding Definition, the Letter B is the angular Point of the Right, Obtuſe, and the Acute-angles, there ſpecified.



7. Two Right-lines, AC, EF, are ſaid to be *parallel*, or *equidiſtant*, when Lines BG, IH, drawn any-where perpendicular to one of them AC, and terminating at the other EF, are equal to each other.

Of Planes, or Surfaces.

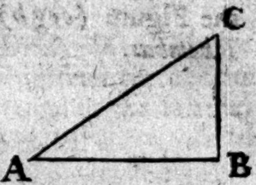
8. A *Figure* is the Form of either a Surface (*viz.* a Superficies), or a Solid.

9. A *Plane-ſurface* is any Figure which lies evenly between its Extremes, or Bounds; and if thoſe Extremes, or Bounds, are Right-lines, the Figure is called a *rectilineal* (or *right-lined*) Plane; but if

the Extremes, or Bounds, of a Plane, are crooked, or Curve-lined, the Figure is then called a *curvilineal Surface*, or *Plane*.

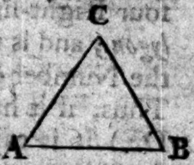
10. Every plane Figure, or Superficies, bounded by three Right-lines, is called a right-lined *Triangle*.

11. A *right-angled Triangle* ABC, is that which has one Right-angle; the Sides AB, BC, containing the Right-angle, are called the *Legs*, and the Side AC, oppoſite the Right-angle, is called the *Hypothenuſe*.

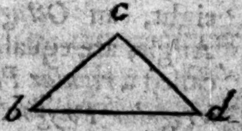


12. When the three Angles are all acute, it is called an *acute-angled Triangle*; if one Angle is obtuſe, the Figure is called an *obtuſe-angled Triangle*.

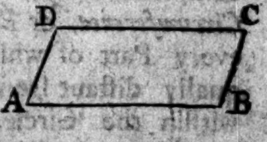
13. An *equilateral Triangle* ABC, has all its Sides equal.

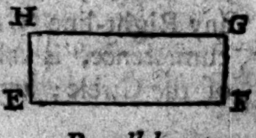


14. An *iſoſceles Triangle bcd*, has two of its Sides equal; and when the three Sides are all unequal, the Figure is called a *ſcalene* Triangle.

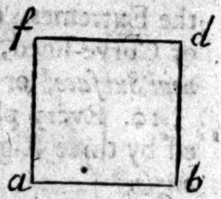


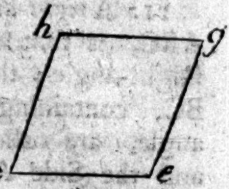
15. Every plane Figure, or Surface, bounded by four Right-lines, is called a *Quadrilateral*: Whereof, thoſe (ABCD) whoſe oppoſite Sides are parallel, are called *Parallelograms*; and thoſe (EFGH) whoſe oppoſite Sides are parallel, and all the Angles right-ones, are called *Rectangles*, or *rectangular Parallelograms*;





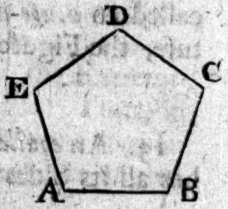
Parallelograms; and if the Sides, as well as the Angles, are all equal, the Figure *(abdf)* is called a *Square*: — When the Sides are all equal, and only the oppoſite Angles equal, the Figure *(cegh)* is called a *Rhombus*.



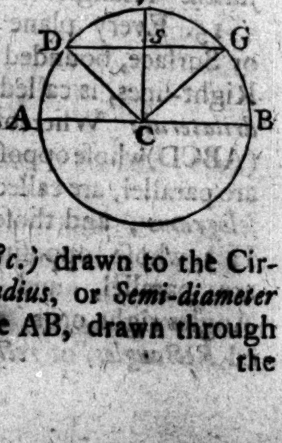


16. Every other plane Figure, bounded by four Right-lines, is called a *Trapezium*.

17. Any plane Figure, or Superficies, bounded by more than four Right-lines, is called a *Polygon*; and is named according to the Number of Sides it contains: Thus, if it has five Sides (ABCDE) it is called a *Pentagon*; if ſix Sides, a *Hexagon*; if ſeven, a *Heptagon*; if eight, an *Octagon*, *&c.* If all the Sides and Angles are equal, as in the Figure (ABCDE), it is called a *regular Polygon*; if otherwiſe, it is called an *irregular Polygon*.



18. A Circle is a plane Figure, bounded by one continued Line, called the *Circumference*, or *Periphery*; every Part of which is equally diſtant from a Point within the Circle, called its Center; from which, any Right-line (CA, CD, *&c.*) drawn to the Circumference, is called the *Radius*, or *Semi-diameter* of the Circle; any Right-line AB, drawn through the



the Center, terminating each Way at the Circumference, is called a *Diameter*; a Right-line DG, leſs than the Diameter, meeting the Circumference in two Points, is called a *Chord*, or *Subtenſe*; and the perpendicular Diſtance *rs*, from the Middle of the Chord to the Circumference, is called a *Verſed-ſine.*

19. A *Segment* of a *Circle* D*r*G, is a Figure bounded by a Part of the Circumference, and its Chord DG; when this laſt is equal to the Diameter of the Circle, the Figure is called a *Semi-circle*; as AD*r*GB.

20. A *Sector* of a *Circle* D*r*GC, is a Figure contained by an Arch (or Arc) thereof, and two Semi-diameters, when theſe two form a Right-angle, or the Arc becomes ¼th of the Circumference, the Figure is called a *Quadrant*, as AD*r*C, (or C*r*GB, ſee the laſt Fig.)

21. The Circumference of every Circle is ſuppoſed to be divided into 360 equal Parts, called *Degrees*, and each Degree into 60 equal Parts, called *Minutes*, each Minute into 60 equal Parts, called *Seconds*, *&c.*

22. Every plane Angle DCG (ſee the preceding Figure) is meaſured by an *Arch* of a *Circle*, contained between the two Lines which form the Angle, and deſcribed upon the angular Point as a Center; and the Quantity of the Angle is eſtimated from the Number of Degrees and Minutes, *&c.* which the ſaid *Arch* contains.

OF SOLIDS.

23. A *Solid* is that which has three Dimenſions, *viz.* *Length*, *Breadth*, and *Thickneſs*: — The Figure of a Solid may be conceived to be generated either by the Motion of a Plane (or Surface) in

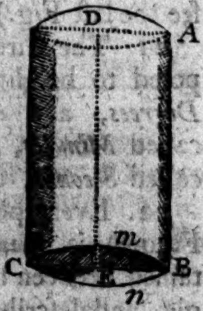
ſome

ſome certain Direction, or by the Revolution of a Plane round ſome Line as an Axis.

24. The *Bounds*, or *Extremes*, of a Solid, are either plane or curved Surfaces.

25. A *Priſm* is a Solid, whoſe two Ends are parallel Planes, of any rectilineal Form whatever; the Planes of the Sides of this Solid are Parallelograms; when theſe ſtand perpendicular to the Plane of the Baſe, the Figure is called an *upright Priſm*; when they ſtand otherwiſe to the Baſe, the Figure is called an *oblique Priſm*; if the two Ends are Parallelograms, the Solid (being then contained under ſix Parallelograms) is called a *Parallelopipedon*; if the ſix bounding Planes are all Rectangles, the Solid is called a *rectangular Parallelopipedon*; and when the ſix bounding Planes are all Squares, the Solid is called a *Cube*.

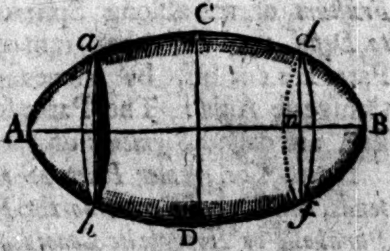
26. A *Cylinder* is a Solid, whoſe two Ends are equal and parallel Circles: This Solid may be conceived to be formed, or generated, either by the Rotation of a Rectangle ABED, about one of its Sides DE, as an Axis; or by the Motion of a Circle C*m*B*n*, in a Direction perpendicular to itſelf, to any aſſigned Altitude DE: This is called a *right Cylinder*. But if the Circle be ſuppoſed to move parallel to itſelf, in any other rectilineal Direction, it will thereby generate an inclined Cylinder; but this Solid never occurs in the Subject of Gauging.



If the two equal Ends of the *Solid* are (inſtead of Circles) of any curvilineal Form whatever, *it* is in general called a *Cylindroid*; and is farther diſtinguiſhed according to the Figure of its Baſes; thus, if the two Ends were two equal, ſimilar, and ſimilarly poſited Ellipſes; that is, the tranſverſe and conjugate Axes of each End, reſpectively parallel to each other; the Solid is then called an *elliptical Cylindroid, &c.*

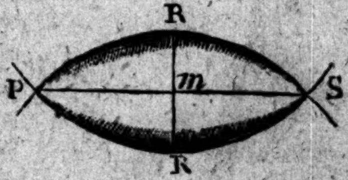
27. A *Pyramid* is a Solid, whereof the Baſe is any right-lined Plane whatever; the Sides of this Solid are plane Triangles, whoſe vertical Angles (*i. e.* thoſe oppoſite the Perimeter of the Baſe) all meet together in a Point above the Baſe, called the *Vertex* of the *Pyramid.*

28. A *Sphere* is a Solid, generated by a Semicircle revolving about its Diameter as an Axis.

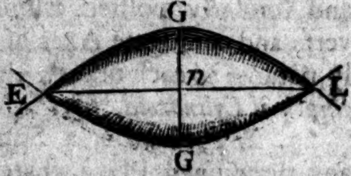


29. A *Spheroid* is a Solid, generated by the Rotation of a Semi-ellipſis about its Diameter as an Axis; if the Rotation be about the tranſverſe Axis AB, the generated Solid ACBD, is called an *oblong Spheroid*; but if the Solid be generated about the conjugate Axis CD, it is called an *oblate Spheroid.*

Note. *The former of the two laſt mentioned Solids, is only applicable to the preſent Subject: Every* ſpheroidical Caſk *reſembling the middle Fruſtum of ſuch a Solid.*



30. A *Parabolic Spindle* is a Solid PRSR, generated by the Rotation of a Parabola PRS*m*, about its Ordinate PS: But if the parabolic Space R*m*S (R*m*P) was to revolve about its Axis R*m*, it would thereby generate a Solid called a *Parabolic Conoid.*



31. An *Hyperbolic Spindle* is a Solid EGLG, generated by the Rotation of an Hyperbola EGL*n*, about its Ordinate EL: But when the *Hyperbolic Space* G*n*L (G*n*E) revolves about its Axis G*n*, it thereby generates the Reſemblance of a Solid, called an *Hyperbolic Conoid*.

32. The *Fruſtum* of a *Cone*,[*] is what remains after a Part is cut off next the Vertex, by a Plane parallel to the Baſe of the Cone: The *middle Fruſtum* of an oblong Spheroid D*ba*C*df* (*ſee Fig. to Defin.* 29.) is what remains after two equal Parts are cut off, by Planes perpendicular to the tranſverſe Axis: The Parts ſo cut off, A*ab* and B*df*, are called *Segments* of the Spheroid.

Note. *The former Part of the laſt Definition extends, with equal Force, to the Fruſtums of Pyramids, Parabolic or Hyperbolic Conoids, and Spindles.*

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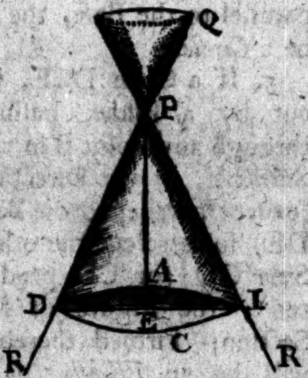
[*] See the Definition of a Cone in the following Page.

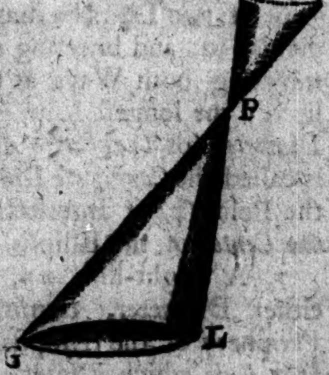
SECTION VI.

Of the Definitions, and some of the principal Properties of the Conic Sections.

Definitions.

1. IN an indefinite Right-line QR, conceive an immoveable (or fixed) Point P; upon which, as a Center, let the ſaid Line be moved juſt round, continually touching the Circumference of a Circle DAIC; placed in any Poſition (except in that of a Plane paſſing through the ſaid fixed Point); then that Part of the Line intercepted between the fixed Point and the Periphery of the Circle, will (by its Rotation) generate the convex Superficies of a Figure called a *Cone*: If the Axis PE, or the Line joining the fixed Point and the Center of the Circle, be perpendicular to the Plane thereof, the





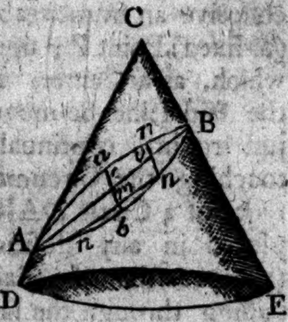
Superficies then deſcribed, will be that of a *right Cone*, as DPI; otherwiſe, it will be an *oblique*, or *ſcalene Cone*, as GPL.

2. The Line PQ (*Fig.* I.) on the contrary Side of the fixed Point P, will alſo generate the convex Surface of another ſimilar Cone; and theſe together are called *oppoſite Cones*.

3. If a right Cone DPI, be cut into two equal Parts, by a Plane perpendicular to that of the Baſe; the Figure of the Section will be a right-lined *iſoſceles Triangle*.

4. If a Cone be cut into two Parts, by a Plane parallel to the Baſe, the Figure of the Section will be a *Circle*.

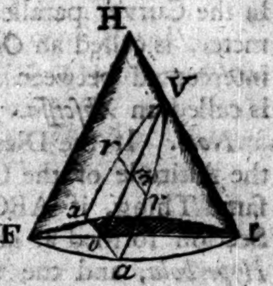
5. If a Cone DCE, be cut by a Plane, paſſing through any Point B in the Side CE, in any Direction (except parallel to the Baſe DE) ſo as to cut the other Side CD (or CD produced); the Figure of the Section, formed thereby, will be an *Ellipſis* (or a *Segment* thereof).



6. If two Lines be drawn in this Figure perpendicular to, and biſecting each other, and each terminating both Ways at the Periphery of the Ellipſis; the longeſt Line AB is called the *Tranſverſe Diameter* (or *Tranſverſe Axis*), and the ſhorteſt *ab*, is called the *Conjugate Diameter* (or *Conjugate Axis*); the Point (*c*) of Interſection of theſe two Lines, is the *Center* of the Ellipſis.

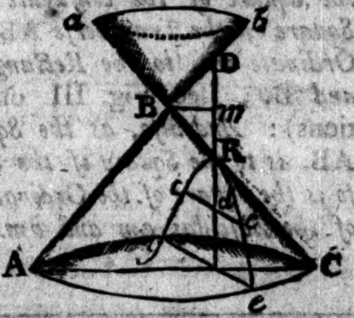
7. A Right-line, *nn*, drawn perpendicular to either Diameter, terminating both Ways at the Periphery of the Ellipſis, is called an *Ordinate* to that Axis which it interſects; and the Diſtance *v*B

(*v*A, *wb* or *ma*) in the Axis, from the Ordinate to the Vertex B (A, *b* or *a*), is called an *Abſciſſa.*



8. If a Cone FHI, be cut into two Parts, by a Plane, in a Direction parallel to the ſlant Side thereof; the Figure of the Section *a*V*av* is called a *Parabola.*

9. The Right-line V*v*, drawn from the Vertex V, parallel to the ſlant Side of the Cone, dividing the Area of the parabolic Section into two equal Parts, is called the *Axis* of the Parabola; any determinate Part from the Vertex V (as V*m*) is called the Abſciſſa; and any Right-line *mr*, drawn perpendicular to the Axis V*v*, terminating at the Curve, is called an *Ordinate.**



10. If a Cone ABC, be cut into two Parts by a Plane, which, being continued, would alſo cut the oppoſite Cone *a*B*b*; the Figure of the Section R*eg* is called an *Hyperbola*:— The Diſtance DR, intercepted between the two oppoſite Cones, is called the *Tranſverſe Diameter* (or *Axis*), and the Diſtance B*m* from the Vertex B, to the Middle of the Tranſverſe, is

---

* Though an Ordinate, ſtrictly ſpeaking, is that Line in a Conic Section, which is biſected by the Axis (or Diameter) terminating each-way at the Curve; yet Geometricians frequently call the Half of this Line (or the Diſtance from the Curve to the Axis, or Diameter) the *Ordinate*: For the general Property of the Curve is the very ſame in both Caſes; becauſe the Squares (or any Power or Multiple) of the Wholes, are in the ſame Ratio, as the Squares (or the ſame Power or Multiple) of their Halves.

called the *Semi-conjugate Diameter* (or *Axis*): Moreover, the Right-line *cd*, drawn from any Point C in the Curve, parallel to the Semi-conjugate Diameter, is called an *Ordinate*; and the Diſtance R*d*, intercepted between that Ordinate and the Vertex, is called an *Abſciſſa*.

Note. If the Diameter of the Baſe be double the Altitude of the Cone, or, which comes to the ſame Thing, if ABC is a Right-angle; then the Section formed as above, is called an *Equilateral Hyperbola*, and the two Diameters DR and twice B*m* become equal to each other.

PROP. I.

The general Property of every Ellipſis will be, as the Square of the conjugate Diameter ab, is to the Square of the Tranſverſe AB, *ſo is the Square of the Ordinate vn, to the Rectangle of the Abſciſſas* A*v* *and* B*v*; (ſee Fig. III. of the preceding Definitions): *And alſo, as the Square of the Tranſverſe* AB, *is to the Square of the conjugate Diameter ab, ſo is the Square of the Ordinate mn, to the Rectangle of the Abſciſſas am and bm.*[*]

PROP.

---

[*] Let DE (*Fig. I. in the Plate*) be the Tranſverſe, and TH the conjugate Diameter of an Ellipſis; through the Center M, and alſo the Point of Interſection (*m*) of the Ordinate (*bb*) with the tranſverſe Diameter, draw RS and GK, each parallel to the Diameter of the Cone's Baſe: Then (ſuppoſing ACE to be a Plane paſſing through the Vertex and the Center of the Baſe of the Cone), by ſimilar Triangles, $DM : RM :: Dm : Gm$; again, by ſimilar Triangles, $EM : SM :: Em : Km$; whence, by multiplying the Antecedents and Conſequents of both Proportions by each other, we get $DM \times EM : RM \times SM :: Dm \times Em : Gm \times Km$; but, by the Property of the Circle, $RM \times SM = MH^2$, and alſo $Gm \times Km = bm^2$; $\therefore DM \times EM : MH^2 :: Dm \times Em : bm^2$; but $DM = EM$, $\because MH^2 : DM^2 (EM^2) :: bm^2 : Dm \times Em$; or, which is the ſame, $TH^2 (4 \times MH^2) : DE^2 (4 \times DM^2) :: bm^2 : Dm \times Em$. Q. E. D.

Draw

PROP. II.

The general Property of every conic Parabola (ſee Fig. IV. of the foregoing *Def.*) *will be, that the Squares of any two Ordinates, r m and a v, are to each other as their correſponding Abſciſſas,* Vm *and* V*v*.†

PROP. III.

The general Property of the Hyperbola (ſee *Fig.* V. of the laſt *Def.*) *will be, as the Square of any Ordinate d c, to the tranſverſe Diameter* DR, *is to the Rectangle contained under the correſponding Abſciſſa* R*d*, *and the Sum of that Abſciſſa and the tranſverſe Diameter*, viz. D*d*; *ſo is the Square of the conjugate Diameter, twice* Bm *to the Square of the tranſverſe* DR.‡

Draw the Ordinate *bn*; then, by the laſt Proportion, we have $bm^2 : EM^2 - Mm^2\ (EM^2 - bn^2$, or $\overline{EM+Mm} \times \overline{EM(MD)-Mm}) :: TH^2 : DE^2$, that is, $bm^2 \times DE^2 = \overline{EM^2 - bn^2} \times TH^2$, or $bn^2 \times TH^2 = EM^2 \times TH^2 - bm^2 \times DE^2$; but $EM^2 \times TH^2 - bm^2 \times DE^2 = 4EM^2 \times TM^2 - bm^2\ (Mn^2) \times DE^2$ (or $4EM2$); whence $bn^2 \times TH^2 = \overline{TM^2 - Mn^2} \times 4EM$ ($= \overline{TM+Mn} \times \overline{TM(MH)-Mn} \times 4EM$); that is, $TH^2 : DE^2\ (4EM^2) :: TM^2 - Mn^2\ (Tn \times Hn) : bn^2$. Q. E. D.

† Draw D*m*G (Fig. II. in the Plate) parallel to the Diameter FI; then the Triangles *v*VI and *m*VG are ſimilar, and ∴ $Vv : vI :: mV : mG$, or $Vv \times mG = Vm \times vI$; but, decauſe $Dm = Fv$, $Vv \times mG \times Dm = Vm \times vI \times Fv$; and, by the Property of the Circle, $mG \times Dm = mr^2$, and alſo $vI \times Fv = va^2$, which being ſubſtituted above, we get $Vv \times mr^2 = Vm \times va^2$; that is, $Vv : Vm :: va^2 : mr^2$. Q. E. D.

‡ Through the Axis *ra* draw M*d*I (Fig. III.) parallel to B*n*, and alſo let the Ordinate *cc* (lying in the ſame Plane with M*d*I) be drawn: Then, by ſimilar Triangles, $Dn : Bn :: Dd : Id$; again, by ſimilar Triangles, $nr : Bn :: dr : dM$, ∴ $Dn \times nr : Bn^2 :: Dd \times dr : Id \times dM$; but, $Dn = nr$, ∴ $nr^2 : Bn^2 :: Dd \times dr : Id \times dM$; moreover, by the Property of the Circle, $cd^2 = Id \times dM$; whence we have $nr^2 : Bn^2 :: Dd \times dr : cd^2$, or, $Dr^2\ (4nr^2) : \overline{2Bn}|^2\ (4Bn^2) :: Dd \times dr : cd^2$. Q. E. D.

The foregoing *Definitions* and *Properties* of the Sections of a Cone are abſolutely neceſſary to be well underſtood by every practical Gauger, who would clearly apprehend what is meant by the Words, *Ellipſis*, *Parabola*, and *Hyperbola*; and alſo by the Solids, which may be conceived to be generated by the Rotation of thoſe *Curves* about their Diameters, or Ordinates: Such as the *Spheroid*, *Parabolic Spindle*, and *Hyperbolic Spindle*, *&c.* from whence the three Varieties of Caſks are formed.

SECTION VII.

OF THE MEASURE OF PLANE FIGURES; *or of finding their Areas* in Ale and Wine Gallons, and Malt Buſhels.*

*THE Area, or Meaſure, of any plane Surface, Geometrically conſidered, is the whole Space contained under the Bounds of the Figure, without any Regard to Thickneſs; as in the Menſuration of *Land*, *Painter's Work*, *&c.* This Area, or ſuperficial Content of the Space, is computed from another Space, of a determinate Form and Magnitude; that is, from a *Square* whoſe Side is one *Inch*, *Foot*, *Yard*, *&c.* called the *meaſuring Unit*; and the Number of ſuch *Squares*, or *Units*, (and Parts of an Unit), that are contained in any plane Figure, is called the *Content*, or *Meaſure* of that Figure. But, in the Subject of Gauging, where the *meaſuring Unit* is one Ale or Wine Gallon, or a Malt Buſhel, it will be moſt commodious, in Order to expreſs the Area of a plane Figure in ſuch Denominations, to conſider the Plane (or rather *Solid)* to be juſt one Inch thick; by which Means, if the Number that expreſſes the ſquare Inches contained in any plane Figure, be divided by the Number expreſſing the cubic Inches in the Ale or Wine Gallon, or Malt Buſhel, we ſhall obtain the Area, or the Content of the Figure, in thoſe Denominations.

Though it is ſaid (above) that any plane Figure will contain a Number of little *equal Squares*, yet it is not to be underſtood that all plane Figures can be *formed* with a Number of ſuch Squares; but becauſe a Square (which can be ſo formed) may be found, whoſe Area ſhall be exactly (or *nearly)* equal to that of any plane Figure whatever.

The very ſame is to be obſerved, with Reſpect to every ſolid Figure (except a *Cube*, or a rectangular *Parallelopipedon)* not being formed with a Number of little *equal Cubes*.

Note. A Gallon of Ale, *(Beer*, or *Vinegar)* contains 282 cubic Inches.

A Gallon of Wine *(Sweets, Cyder, Perry, Verjuice, Waſh, Low Wines, Spirits, &c.)* contains 231 cubic Inches.

A *Wincheſter* Buſhel of Malt contains 2150.42 cubic Inches.

		Gall.	
In 1 Barrel	*London* Beer are	36	each Gallon 282 cubic Inches.
	London Ale are	32	
	Country, Beer & Ale	34	
	Vinegar	34	
	1 Barrel of Sweets	31½	Wine Measure.

Prop. I.

To find the Area of any plane Triangle, in Ale and Wine Gallons, and Malt Buſhels.

Rule.

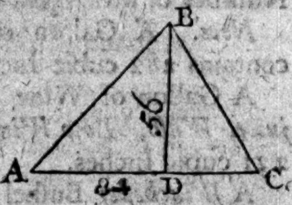
RULE.

From any one of the given Angles let fall a Perpendicular upon the Side oppoſite (produced if needful); multiply this Side, taken in Inches, by half the Perpendicular, taken in the ſame Meaſure, the Product will be the Area of the Triangle in ſquare Inches; which being divided by 282 for Ale, 231 for Wine Gallons, and by 2150.42 for Malt Buſhels, the Quotient will be the required Area of the Triangle.

Note. The Meaſure, or ſuperficial Content, of any plane Triangle is likewiſe obtained, by multiplying the whole Perpendicular by half the Baſe; or by taking half the Product contained under the whole Baſe and Perpendicular.

EXAMPLE.

The Baſe AC, of the Triangle ABC (or AEC) is 84, and the Perpendicular BD (or EF) is 56 Inches; required the Area in Ale Gallons, *&c.*



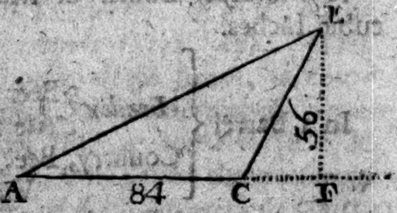
OPERATION.

Baſe AC 84

½ Perp. BD(EF) 28

672

168



Product 2352, the Area of the Triangle ABC [(AEC) in Inches.

282

282)2352.00(8.34, the Area in Ale Gallons.
231)2352.00(10.18, the Area in Wine Gallons.
2150.42)2352.00(1.09, the Area in Malt Buſhels.

By the Sliding-Rule.

$$\text{To}\left\{\begin{array}{c}282\\231\\2150.42\end{array}\right\}$$ on A, ſet 28 (or 84) on B; then oppoſite 84 (or 28) on the firſt Radius on A, is the above Areas reſpectively on B.*

Note. The above Areas may be obtained by the Lines D and C, but not without extracting the Square Root, which would render the Operation more troubleſome than that above exhibited, by the Lines A and B.

Prop. II.

To find the Area of a Square afdb, in Ale and Wine Gallons, and Malt Buſhels. (See Defin. 15, P. 56.)

Rule.

---

* It is evident that the Characteriſtics (or Indices) in Logarithms, anſwer to the Number of Radii of the Rule; that is, any Number greater than 1 and leſs than 10, where the Characteriſtic is = 0, is found on the firſt Radius; likewiſe any Number greater than 10 and leſs than 100, the Characteriſtic being = 1, muſt be found (according to the *true* Numeration of the Lines) on the ſecond Radius, &c. For the Logarithm of any Number, is the very ſame as the Log. of 10, 100, 1000, &c. times that Number, except in the Characteriſtics, which are = the Exponents of the Powers of 10: Therefore, in the preceding Example, the Log. 2.8 + Log. 8.4 — Log. 2.82 (= Log. 28 + Log. 84 — Log. 282) = Log. 8.34: Whence it appears, that by ſetting 2.8 on B to 1 on A, we ſhall get the Diſtance from 1 to 2.8 on B, and from 1 to 8.4 on A in one Sum, meaſured on the Slide B; but this Diſtance muſt evidently be diminiſhed by *that* denoting the Log. of 2.82; to effect which, move the Slide towards the right Hand, 'till 2.8 on B, ſtands oppoſite 2.82 on A; then againſt 8.4 on A, is 8.34 on B, the ſame as before.

RULE.

Multiply the Side of the Square by itſelf, and divide the Product by 282 for Ale, 231 for Wine Gallons, and by 2150.42 for Malt Buſhels; or (which will be exact enough in Practice) by 2150.

EXAMPLE.

The Side of the Square *afdb*, being 50 Inches; required its Area, in Ale Gallons, *&c.*

OPERATION.

$$\begin{array}{r} 50 \\ 50 \\ \hline \end{array}$$

Product 2500 the Area in Inches: Then
282)2500.00(8.86, the Area in Ale Gallons.
231)2500.00(10.82, the Area in Wine Gallons.
2150)2500.00(1.16, the Area in Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{matrix} 16.79 \\ 15.19 \\ 46.37 \end{matrix}\right\}$ on D, ſet 1 on C; and oppoſite 50 on D, is $\left\{\begin{matrix} 8.86 \\ 10.82 \\ 1.16 \end{matrix}\right\}$ on C, the ſame as above.

Otherwiſe, by the Sliding-Rule.

To $\left\{\begin{matrix} 282 \\ 231 \\ 2150 \end{matrix}\right\}$ on A, ſet 50 on B; and againſt 50 on A, is the above Areas, reſpectively, on B.

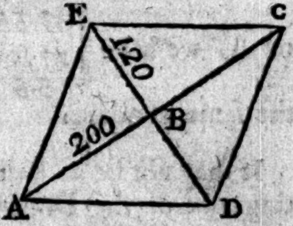
PROP.

PROP. III.

To determine the Area of a Rhombus, in Ale and Wine Gallons, and Malt Buſhels.

RULE.

Multiply the two Diagonals together; Half that that Product divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.*



EXAMPLE.

Suppoſe the Diagonal ED 120, and AC 200 Inches; required the Area of the Rhombus in Ale Gallons, *&c.*

OPERATION.

120
200

The Product of the Diag. 24000

Half the Product is 12000 equal to the Area [in Inches.

282)

---

* By Reaſon of the equal and parallel Lines, it is plain (*Eu.* 5. & 29. 1.) that the oppoſite Angles are all biſected by the Diagonals; conſequently the Rhombus (*ſee Fig. to Prop.* III.) is evidently divided (by the Diagonals) into four right-angled Triangles, ſimilar and equal in every Reſpect; the Area of any one of which will be expreſſed by $AB \times \frac{BD}{2}$, or $\frac{AC}{2} \times \frac{ED}{4}$; conſequently the Area of the whole Rhombus is $= \frac{AC \times ED}{2}$. Q. E. D.

282)12000.00(42.55 equal to the Area in Ale Gallons.

231)12000.00(51.94 = the Area in Wine Gallons.

2150)12000.00(5.58 = the Area in Malt Buſhels.

By the Sliding-Rule.

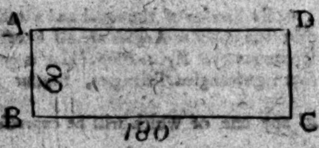
To $\left\{\begin{matrix}282\\231\\2150\end{matrix}\right\}$ on A, ſet 120 on B; then againſt 100 on A, we have $\left\{\begin{matrix}42.55\\51.94\\5.5\end{matrix}\right\}$ the required Areas on B, the ſame as above.

It may not be amiſs to obſerve here, that by ſetting one of the Dimenſions on B, to 2150 on A, the other Dimenſion cannot be found on the Line A; unleſs the Rule conſiſts of more than two Radius's, or the ſaid Dimenſion is equal to, or greater than 100: We muſt therefore, in ſuch Caſes, have Recourſe to the Method already delivered at *Page* 42.

PROP. IV.

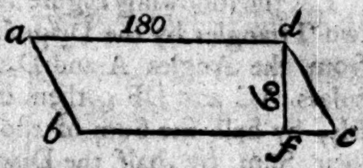
To find the Area of a Parallelogram, in Ale and Wine Gallons, and Malt Buſhels.

RULE.



If it is a rectangular Parallelogram, as ABCD: Multiply the longeſt Side by the ſhorteſt; if otherwiſe, as *abcd* (called by Some a *Rhomboides*); then multiply the longeſt Side *bc*, *bc*, by the Perpendicular *df*, and divide the Product by 282, 231, and 2150; the Quotients will be the Areas in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE.



Suppoſe the longeſt Side BC (or *bc*) 180, the ſhorteſt Side AB (or the Perpendicular *df*) 60 Inches; required the Area in Ale Gallons, &c.

OPERATION.

180
60
―――

Product is 10800, equal the Area in Inches.

282)10800.00(38.3 *nearly*, = the Area in Ale Gallons.

231)10800.00(46.75 = the Area in Wine Gallons.

2150)10800.00(5.02 = the Area in Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, ſet either of the above given Dimenſions on B; then oppoſite the other Dimenſion on A, is $\left\{\begin{array}{c}38.3\\46.75\\5.02\end{array}\right\}$ on B, the ſame as above.

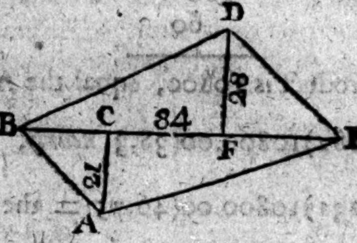
PROP. V.

To find the Area of a Trapezium, in Ale and Wine Gallons, and Malt Buſhels.

RULE.

Let the following Figure ABDE, be divided into two Triangles by the Diagonal EB; upon which, from the Angles A and D, let fall the Perpendiculars AC and DF; then multiply the Diagonal BE, by half the Sum of thoſe Perpendiculars; or their Sum by half the Diagonal, and divide the Product by 282 for Ale, 231 for Wine, and 2150 for Malt Buſhels.

EXAMPLE.



Suppoſe the Diagonal BE to be 84 Inches, the Perpendiculars AC and DF to be 21 and 28 Inches reſpectively; required the Area in Ale Gallons, *&c.*

OPERATION.

28
21

Sum of the Perp. 49
½ the Diagonal (84) is 42

98
196

Product is 2058, the Area in Inches.

282)

282)2058.0(7.3 *nearly*, the Area in Ale Gallons.
231)2058.0(8.9 Wine Gallons.
2150)2058.00(.95 Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, ſet 49 on B; then oppoſite 42 on A, we have $\left\{\begin{array}{c}7.3\\8.9\\0.95\end{array}\right\}$ on B, the ſame as above.

By the preceding Method of dividing the Trapezium, the Meaſure of any irregular Polygon may very eaſily be obtained: For if the whole Figure is divided into Triangles, and the Area of each of thoſe be found (*by Prop.* I.), the Sum of which will be the Meaſure of the whole Polygon.

PROP. VI.

To find the Area of any regular Polygon, in Ale and Wine Gallons, and Malt Buſhels.

RULE.

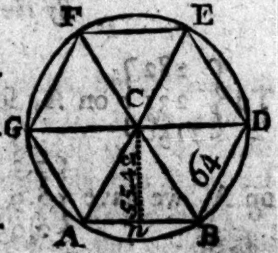
Half the Sum of all the Sides, being multiplied by a Line drawn from the Middle of any one of the Sides to the Center of the Polygon (or the Circle circumſcribing it), and the Product divided by 282, 231, and 2150; the Quotients will be the Areas in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE.

In the Hexagon ABDEFG, if one of its Sides AB (BD, *&c.*) be 64 Inches, the Perpendicular

Cn (Cr, &c.) will be 55.42 Inches; required the Area in Ale Gallons, &c.

OPERATION.



64
6

384

½ the Sum of the Sides 192
Perpendicular Cn 55.42

384
768
960
960

Product 10640.64, the Area in Inches.

282)10640.64(37.73 = the Area in Ale Gallons,
231)10640.64(46.06 Wine Gallons.
2150)10640.64(4.94 Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, ſet 192 on B; then againſt 55.4 on A, we have $\left\{\begin{array}{c}37.73\\46.06\\4.94\end{array}\right\}$ on B, the ſame as above.

Any regular Polygon is compoſed of as many iſoſceles Triangles as it contains Sides, and may be inſcribed in a Circle, whoſe Center is that of the Polygon's; whence the *(equal)* Angles at the Center become known: And therefore it follows, that, if the Meaſure of the Side of any regular Polygon be one Inch, the Perpendicular

C*n* (ſee the laſt Figure) and the Area of the Polygon (in Inches) become known, which being divided by the proper Diviſors, the Quotients will give the reſpective Multipliers* for Ale and Wine Gallons, and Malt Buſhels: — Theſe Multipliers, conformable to all Authors on this Subject, I have exhibited in the following Table, for ſix different Kinds of regular Polygons. Now as it is well known, to Geometricians, that the Areas of ſimilar (or like) plane Figures, are in Proportion to one another, as the Squares of their correſponding Sides; therefore, having obtained (as above) the Area of a Polygon whoſe Side is Unity, we then ſay, by Proportion, as the Square of 1 (which is 1), is to the Square of the Side of the Polygon whoſe Area is ſought, ſo is the Area of that Polygon whoſe Side is Unity (expreſſed in the following Table), to the Area of the Polygon ſought; Hence the following

Rule.

---

* Theſe Multipliers, or Factors, may be otherwiſe derived, by ſuppoſing (inſtead of the Side) the Radius of the Circle circumſcribing the Polygon $= 1$: For if a denote the Number of Sides of a Polygon circumſcribing that Circle, $t =$ the Tangent of $\frac{1}{2}$ the Angle at the Center; which Angle is always known, from the Number of Sides of the circumſcribing Polygon: Then the Area of the Polygon circumſcribing the Circle whoſe Radius is $= 1$, will be expreſſed by $1 \times t \times a$, or ta, and the Square of one of its Sides by $4t^2$; $\therefore 4t^2 : ta :: r^2$ (the Square of the Side of any other Polygon) : its Area; but $\frac{ta}{4t^2}$ (or $\frac{a}{4t}$) is $=$ the tabular Number, or Factor (ſee *Shirtcliffe's Gauging*, P. 68.), and is therefore $=$ the Area of a Polygon, whoſe Side is Unity, and Number of Sides $= a$: Which is thus proved:—Let m repreſent the Sine of an Angle whoſe Tangent is t; then, by ſimilar Triangles, $t : 1$ (Radius) $:: m : \frac{m}{t} =$ the Coſine, $\therefore \frac{m^2 \times a}{t} =$ the Area of the Polyogn whoſe Side is $2m$, and Number of Sides a; whence, by ſimilar Figures, $4m^2 : \frac{m^2 \times a}{t} :: \overline{1}^2 : \frac{m^2 \times a}{4m^2 \times t} = \frac{a}{4t}$. Q. E. I.

RULE.

Multiply the Square of the Side of a given regular Polygon by ſuch a Number, taken out of the following Table, as is agreeable to the Name of the Polygon; and the Product will be the Area thereof, in the ſame Denomination as the Factor that was made Uſe of.

A TABLE *for finding the* AREAS *of regular* POLYGONS.

The Names of the Polygons.	Numb. of Sides.	The Angle at the Center.	Area in Inches when the Side of the Polygon is Unity.	The Area in Ale Gallons.	The Area in Wine Gallons.	The Area in Malt Buſhels.
Pentagon	5	72° 0′	1.7204	.006099	.007445	.00078
Hexagon	6	60 0	2.5980	.009212	.011246	.001208
Heptagon	7	51 25$\frac{5}{7}$	3.6339	.012883	.015727	.001689
Octagon	8	45 0	4.8284	.01712	.0209	.002245
Nonagon	9	40 0	6.1818	.021925	.026726	.002875
Decagon	10	36 0	7.6942	.027287	.033311	.003579

In the preceding Example the Side of the Hexagon is 64 Inches, which being multiplied by 2.598, the tabular Number for that Figure, gives 10641.4, the Area in Inches, the ſame as at *Page 76, very nearly*.

Having ſhewn the Methods of computing the Areas of ſuch right-lined Planes, as chiefly occur in the Practice of Gauging; I ſhall now proceed to determine the Areas of curvilineal Planes; as the Circle, Ellipſis, and their Segments, *&c.* But, firſt of all, it will be very neceſſary to ſhew the Learner, how to find the Circumference of a Circle, by having its Diameter given, and the contrary.

It is now looked upon, even by Mathematicians of the firſt Rank, as abſolutely impoſſible to determine the *exact* Proportion of the Diameter and Circumference of a Circle.

That *great* Geometer *Archimedes*, about two Thouſand Years ago, firſt diſcovered this Proportion to be nearly as 7 to 22; that is, if the Diameter of a Circle be 7, its Circumference will be 22, *very nearly*.

Since *Archimedes*'s Time, various Methods have been invented, whereby the ſaid Proportion may be approximated to a very great Degree of Exactneſs. *Van Ceulen* (a *Dutch* Man) found, by incredible Pains, that if the Diameter of a Circle be repreſented by 1, the Circumference thereof will be 3.14159265358979323846264338327950288, *extremely near*; for if the laſt Decimal Figure be ſuppoſed 9, the ſaid Number (3.1415, *&c.*) would then exceed the *true* Circumference of a Circle whoſe Diameter is 1: — This laſt Number was not only confirmed, but was extended to double the Number of Decimal Places, by that ingenious and moſt indefatigable Mathematician, the late Mr. *Abr. Sharp*, of *Little Horton*, near *Bradford*, in *Yorkſhire*.

But, in the ordinary Practice of Gauging, it will be unneceſſary to take any more than 3.14159 (or 3.1416): Hence it is evident, that if the Diameter of any Circle be multiplied by 3.14159, the Product will be the Circumference of that Circle; *very nearly*.

EXAMPLE.

To find the Circumference of a Circle, whoſe Diameter is 40 Inches.

OPERATION.

Operation.

$$\begin{array}{r} 3.1416 \\ 40 \\ \hline \end{array}$$

Product 125.6640 = the Circumference in Inches.

By the Sliding-Rule.

Set 1 on B, to 3.1416 on A, then againſt 40 on B, is 125.6 on A.

It is evident, from this Example, that if the Circumference of any Circle be divided by 3.1416, the Quotient will be the Diameter thereof, *very nearly*.

If the Diameter and Circumference of a Circle are known, its Area will be found by multiplying half the Circumference by half the Diameter.*

But ſince the Areas of Circles (as well as all other ſimilar plane Figures) are in Proportion to one another, as the Squares of their Diameters (or like Dimenſions); it follows, that, if we have the the Area of a Circle whoſe Diameter is Unity, we can eaſily obtain the Area of any Circle, whoſe Diameter is given, without finding its Circumference at all: Suppoſe, for Example, the Diameter of a Circle to be 1; then the Circumference, by the foreſaid Proportion, will be 3.1416 *very near*; therefore, by the preceding Rule, we have the following

---

* This evidently follows from the Rule given for regular Polygons, *Page* 35: For, ſince that Rule is general, let the Number of equal Sides be what it will; it follows, by conceiving a Polygon of an *indefinite* Number of Sides, that the Perpendicular Ca (ſee Fig. on P. 76) will then become the Radius of the Circle circumſcribing that Polygon *indefinitely near*, and conſequently the Perimeter of ſuch a Polygon is, *very nearly*, equal to the Periphery of its circumſcribing Circle; whence it is evident, that the Meaſure of any Circle is equal to a Rectangle contained under half its Periphery and half its Diameter.

OPERATION.

3.1416

½ the Circumference 1.5708
½ the Diameter .5

Product is .78540, the Area of a Circle whoſe Diameter is 1, *nearly*.

Therefore if the Square of the Diameter of any Circle be multiplied by .7854, the Product will be the Area, or Meaſure, of the Circle, in that Denomination whereby the Diameter was expreſſed, whether *Inches*, *Feet*, *Yards*, *&c.*

As, for Inſtance, ſuppoſe the Diameter of a Circle be 30 Inches, the Square whereof is 900; this being multiplied by .7854, gives 706.86 ſquare Inches, the Area ſought, *nearly*.

Now if the Area of any Circle in Inches (and in all other Figures) be divided by the Number of cubic Inches contained in a Gallon of Ale or Wine, *&c.* we ſhall obtain the Area of the Circle in thoſe Denominations: But, in order to avoid the above troubleſome Multiplier (.7854), in finding the Area of a Circle in Ale or Wine Gallons, or Malt Buſhels, we need only to ſquare the Diameter, and multiply that by the Quotients of .7854 divided by the reſpective Diviſors; or elſe divide the Square of the given Diameter, by the Quotients of the reſpective Diviſors for Squares divided by .7854; and the Products, or Quotients, will be the Area of a Circle in the ſame Denomination as that of the Factor, or Diviſor, uſed.

Diviſors for Squares, *&c.* — **Factors for Circles, *&c.***

282) .78539 *&c.* (.0027851 *&c.* Ale Gallon.
231) .78539 *&c.* (.003399992 Wine Gallon.
2150) .79539 *&c.* (.0003652 Malt Buſhel.

Diviſors for Circles.

.785398) 282.00 *&c.* (359.05 Ale Gallons.
.785398) 231.00 (294.118 Wine Gallons.
.785399) 2150.420 (2738 Malt Buſhels.

After the very ſame Manner may the Factors (and Diviſors) be found, for obtaining the Areas of Circles in any other Denominations; which, for the Sake of Brevity, I ſhall exhibit in the following Table.

The Factors, or Multipliers, for finding the Areas of Squares, *&c.* in Gallons, Buſhels, *&c.* are obtained by dividing Unity by the Diviſor for Squares, *&c.* in the ſame Denomination.

Diviſors for Squares. — **Factors for Squares.**

282) 1.000000 (.003546 Ale Gallons.
231) 1.000000 (.004329 Wine Gallons.
2150.42) 1.000000 (.000465 Malt Buſhels.

Note. The above Factors for Circles may otherwiſe be obtained, by dividing Unity by the reſpective Diviſors for Circles in Ale and Wine Gallons.

The Gauge-points (on the Line D) on the Sliding-Rule, for Ale and Wine Gallons, and Malt Buſhels, are the ſquare Roots of the Diviſors for Squares, or Circles, in Ale and Wine Gallons and Malt Buſhels, as follow.

Diviſors for Squares, *&c.* $\left\{\begin{array}{c}282\\231\\2150.42\end{array}\right\}$, the Square Roots are

$\left\{\begin{array}{c}16.79\\15.19\\46.37\end{array}\right\}$ the Gauge-points for Squares.

Diviſors for Circles, &c. $\left\{\begin{array}{c}359.053\\294.118\\2737.92\end{array}\right\}$, the Square Roots are $\left\{\begin{array}{c}18.95\\17.15\\52.32\end{array}\right\}$ the Gauge-points for Circles.

The above Gauge-points are manifeſtly the Sides of Squares, and the Diameters of Circles, whoſe Areas are one Ale or Wine Gallon, or Malt Buſhel.

A TABLE *of* Multipliers (or Factors), Diviſors, *and* Gauge-points, *for* Squares *and* Circles.

The Side of a Square, or the Diameter of a Circle, is 1.	Multipliers for Squares, &c.	Multipliers for Circles, &c.	Diviſors for Squares.	Diviſors for Circles.	Gauge-points for Squares	Gauge-points for Circles.
is 1.	1	.785398	1	1.27324	1	1.128
Ale Gallon	.003546	.0027851	282	359.05	16.79	18.95
Wine Gallon	.004329	.00339999	231	294.12	15.19	17.15
Wincheſter Malt(orCorn) Buſhel . .	.000465	.000365	2150.42	2738.00	46.37	52.32
Malt(orCorn) Gallon .	.00372	.002922	268.8	342.24	16.39	18.5
A Pound of neat Tallow . . .	.031844	.025101	31.4	39.98	5.60	6.32
A Pound of hard Soap	.036845	.028939	27.14	34.56	5.209	5.88
A Pound of green ſoft Soap . . .	.038956	.0306	25.67	32.68	5.06	5.72
A Pound of white ſoft Soap . . .	.0391235	.030731	25.56	32.54	5.05	5.7
A Pound of green Starch	.028735	.022563	34.8	44.32	5.9	6.65
A Pound of dry Starch	.024563	.019493	40.3	51.3	6.34	7.16

Prop. VIII.

Having given the Diameter of a Circle; to find its Area in Ale and Wine Gallons, and Malt Buſhels, &c.

Rule.

RULE.

Let the Square of the given Diameter be multiplied, or divided, by a Multiplier, or a Diviſor, agreeable to that Denomination, in which the Area of the Circle is required; and the Product, or Quotient, will be the ſaid Area ſought.

EXAMPLE.

Suppoſe the Diameter of a Circle 68 Inches, required its Area in Ale and Wine Gallons, and Malt Buſhels.

OPERATION.

$$\begin{array}{r} 68\\ 68\\ \hline 544\\ 408\\ \hline \end{array}$$

The Square of the Diam. is 4624, which being multiplied (ſee the preceding Table) by

$$\left\{\begin{array}{l} .0027851 \text{ for Ale}\\ .0034 \text{ for Wine, and}\\ .000365 \text{ for Malt Buſhels} \end{array}\right\}, \text{ or divided by}$$

$$\left\{\begin{array}{l} 359.05 \text{ for Ale}\\ 294.12 \text{ for Wine, and}\\ 2737.92 \text{ for Malt Buſhels} \end{array}\right\}, \text{ the Products, or}$$

Quotients, give 12.87, 15.72, and 1.68, for the required Area of the Circle, in Ale and Wine Gallons, and Malt Buſhels reſpectively.

By the Sliding-Rule.

To $\left\{\begin{matrix}18.95\\17.15\\52.32\end{matrix}\right\}$ marked $\left\{\begin{matrix}A.G\\W.G\\M.R\end{matrix}\right\}$ on D, ſet 1 on C; then againſt 68 on D, is $\left\{\begin{matrix}12.87\\15.72\\1.68\end{matrix}\right\}$ on C, the ſame as before.

PROP. IX.

Having given the Length of the Arch, and the Semi-diameter (or Radius) of the Circle; to find the Area of the Sector, in Ale and Wine Gallons, and Malt Buſhels.

RULE.

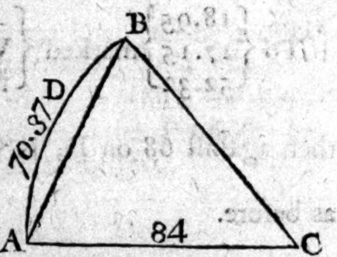
Multiply half the Length of the Arch by the Semi-diameter of the Circle; and divide the Product by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

EXAMPLE.

Let ADBC repreſent a Sector of a Circle, whoſe Semi-diameter AC (or BC) is 84 Inches, and the Arch ADB is 70.4 Inches; required the Area in Ale Gallons, *&c.*

OPERATION.

OPERATION.



½ the Arch ADB 35.185
Semi-diameter AC 84

140740
281480

Area in Inches 2955.540

282)2955.54(10.48 Ale Gallons.
231)2955.54(12.8 Wine Gallons, *nearly.*
2150)2955.54(1.37 Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, ſet 84 on B; then oppoſite 35.18 on A, we have $\left\{\begin{array}{c}10.48\\12.8\\1.37\end{array}\right\}$ on B, the above required Areas.

But if the Arch ADB, or the Meaſure of the Angle ACB, in Degrees and Minutes is given, and likewiſe the Semi-diameter AC: Then multiply the Number of Degrees and Minutes (reduced to the Decimal Parts of a Degree) by the Square of the given Semi-diameter, and that Product by .000030945 for Ale, .000037777 for Wine Gallons, and by .000004059 for Malt Buſhels.

In the preceding Example, the Arch ADB, or the Angle ACB, is found to be 48 Degrees, *nearly*; then 7056 (the Square of 84) being multiplied by 48, and that Product (which is 338688) by .000030945, gives 10.48 Ale Gallons: Moreover, the above Product (338688) being multiplied by .000037777 gives 12.8 Wine Gallons; and if 338688 be multiplied by .000004059, the Product will be 1.37 Malt Buſhels, the very ſame as before.[*]

By the laſt Propoſition, the Area of the Segment of a Circle may be obtained: — For if the Area of the Triangle ABC, be ſubtracted from *that* of the Sector ADBC, there will remain the Area of the Segment ADBA: But ſince it is very troubleſome to get the Length of the Arch of a Segment of a Circle; I ſhall therefore give one general Rule, whereby the Area of that Figure may be found to a very great Degree of Exactneſs, by having only its Chord and verſed Sine given; from whence the Diameter of the Circle is very eaſily obtained, by either of the following Methods.

If the Sum of the Squares of the Semi-chord and verſed Sine, be divided by the verſed Sine, the Quotient will be the Diameter of the Circle, to which that Segment correſponds:—Or, if the Square of the Semi-chord be divided by the verſed Sine, and the Quotient thereof added to the verſed Sine, that Sum will likewiſe give the required Diameter.

---

[*] If the Diameter of a Circle be $= 2$, its Circumference will be $= 6.2831$, &c. and therefore $\frac{6.2831}{360}$ ($= .017453$, &c.) will expreſs the Length of the Arch of one Degree, when the Radius of the Circle is 1:—Now let b denote any Number of Degrees and Minutes, &c. reduced to the Decimal Parts of a Degree; then will $.017453 \times b$ repreſent the Length of thoſe Degrees, &c. in the ſame Meaſure of which the Radius is 1; then, becauſe ſimilar Arcs (as well as the whole Peripheries) of unequal Circles, are to one another as their Radii, we have 1 (the Radius of the leſſer Circle) $: b \times .017453 :: s : bs \times .017453$, the Length of the Arch to the Radius s; $\therefore bs \times \frac{.017453}{2} \times s$, or $.00872664 \times bs^2 =$ the Content of the Sector in Inches; conſequently the Content in Ale Gallons is $= \frac{.00872664}{282} \times bs^2 = .000030945 \times bs^2$.

Suppoſe, for Example, the Chord of a Segment of a Circle be 24, and its verſed Sine 8; required the Diameter of that Circle.

OPERATION.

```
                      Semi-chord 12
                                 12
                              --------
                                144
Add the Squ. of the V. Sine      64
                              --- ----
                              8)208(26, the required Diameter.
                                16
                                ----
                                 48
                                 48
                                ----
                                 ..
```

Otherwiſe, by the ſecond Method.

The Square of the Semi-chord is 144, which being divided by the verſed Sine (8) gives 18, to which add the verſed Sine, and we have 26, the required Diameter, as above.

Both theſe Methods are very eaſily derived from the Properties of the Circle, which are well known to Geometers.

PROP. X.

Having given the Chord and verſed Sine of the Segment of a Circle; to find its Area in Ale and Wine Gallons, and Malt Buſhels.

RULE.

RULE.*

Divide the Difference between the verſed Sine and the Semi-diameter by 4, and *note* the Quotient.

1. Subtract the Square
2. Subtract four times the Square
3. Alſo take nine times the Square

} of the above *noted* Quotient, from the Square of the Semi-diameter, and *note* the Difference.

Then to four times the Sum of the Square Roots of the firſt and third Differences, add twice the Square Root of the ſecond; to this Sum add the Semi-diameter and Semi-chord.

Multiply that Total by $\frac{1}{6}$th Part of the Difference between the Semi-diameter and the verſed Sine; this Product being taken from 1.57079 times the Square of the Semi-diameter, leaves the Meaſure of the Segment in Inches; which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

EXAMPLE.

Required the Area of the Segment of a Circle, whoſe Chord is 40, and the verſed Sine 10 Inches.

---

* This Rule is very eaſily deduced, from the Method of equidiſtant Ordinates explained farther on.

OPERATION.

The Square of the Semi-chord is 400
The Square of the versed Sine is 100

10) 500 (50 = the Diameter of the Circle.

¼th of the Difference between the Semi-diameter (25) and the versed Sine (10) } 3.75

The Square of 3.75 is 14.0625, which being multiplied by 4, gives 56.25; alſo 14.0625 being multiplied by 9, gives 126.5625; then the Square of the Semi-diameter (25) being 625; therefore

1. From 625 ſubtract 14.0625, the Remainder is 610.9375.

2. From 625 ſubtract 56.25, and there remains 568.75; and

3. From 625 take 126.5625, and there remains 498.4375.

The Square Root of the	1st	Difference is	24.7171
	2d		23.8484
	3d		22.3257

Four times the Sum of the 1st and 3d is	188.1712
Twice the 2d Square Root	47.6968
The Semi-diameter	25
The Semi-chord	20
Total	280.8680
Multiply by ⅙th of the Diff. between the Semi-diameter and verſed Sine . . .	2.5
	14043400
	5617360
Product	702.17000

The Square of 25 (the Semi-diameter)
multiplied by 1.57079 is 981.74
Subtract the above Product 702.17

The required Area of the Segment in Inches, *nearly* . . . . . } 279.57

282)279.57(.99, Area in Ale Gallons.
231)279.57(1.21 Wine Gallons.
2150)279.57(.13 Malt Buſhels.

It may not be amiſs to inform the Reader, that the greateſt Error which can ever happen in computing, by the preceding Rule, the Area of any Segment of a Circle, will not exceed the *true* Meaſure $\frac{1}{600}$th Part of the Whole; and in many Circumſtances, eſpecially when the Segment approaches near to a Semi-circle, will be more exact than by any *general Rule* I have hitherto met with.

But the moſt expeditious Method of computing the Meaſure of the Segment of a Circle is, by a Table which is formed of the Areas of the Segments of a Circle,* whoſe Diameter is 1, and which is

---

* It may not be improper to ſhew an eaſy Method of making this Table of the Areas of the Segments of a Circle; and likewiſe the Reaſon of finding, thereby, the Area of a Segment of any Circle, by having only the Diameter thereof, and the verſed Sine of the Segment given: — This laſt depends on the following

Theorem.

From the common Center of any two concentric Circles, let two Right-lines be drawn to any two Points in the outermoſt Circumference, and alſo let the Chord of the Arc of each Circle, included by thoſe two Lines, be drawn: Then will the verſed Sines of the Segments ſo formed, be to each other in the Ratio of the correſponding Radii (or Diameters) of the two Circles; and the Areas of thoſe Segments will be as the Squares of thoſe Radii, or Diameters.

Let OB and OD (Fig. IV.) be the two Right-lines drawn from the common Center O; draw the Chords *db* and DB, and perdendicular thereto draw the Radius OC; moreover let the Chords *cb* and CB be drawn. Then, by ſimilar Triangles, we have, *cb* : CB :: O*b* : OB, and alſo *cb* : CB :: *cn* : C*m*; whence, by Equality, O*b* : OB :: *cn* : C*m*; or *ce* (2O*b*) : CE (2OB) :: *cn* : C*m*.

ſuppoſed to be divided, by Chords perpendicular thereto, into 1000 equal Parts: For if the verſed Sine

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Moreover, becauſe ſimilar Arcs of unequal Circles are as their correſponding Radii, it will be, as O*b* : the Arc *bec* :: OB : the Arc BFC, and (by 15. *Eu.* 5.) as $Ob \times Ob$: the Arc $bec \times Ob$ (:: OB : Arc BFC) :: $OB \times OB$: the Arc $BFC \times OB$, that is Ob^2 : OB^2 :: the Sector O*dcb* : the Sector GDCB; but (by 19. *Euclid* 6.) Ob^2 : OB^2 :: the Triangle O*db* : the Triangle ODB; whence, by Equality, the Sector O*dcb* : the Triangle O*db* :: the Sector ODCB : the Triangle ODB; therefore, by Diviſion, O*dcb*—O*db* : O*db* :: ODCB—ODB : ODB, or the Segment *dcb* : the Seg. DCB (:: O*db* : ODB) :: Ob^2 : OB^2 (:: ce^2 : CE^2). Q. E. D.

A Method of computing the Table of the Areas of Segments of a Circle.

Suppoſe the Radius of a Circle = 1; then will the Meaſure of any Segment thereof, be expreſſed by $\frac{1}{2}$ the Meaſure of the Arc of that Segment, *minus* $\frac{1}{2}$ the Sine of *that* Arc: For it is evident, the former ($ceb \times 1$, Fig. IV.) expreſſes the Meaſure of the Sector O*dcb*, and the latter ($\frac{1}{2}bf \times 1$) the Meaſure of the Triangle O*db*.

Now, in order to determine the Meaſure of the Segment of a Circle to any propoſed verſed Sine, ſuppoſing the Radius = 1, and divided into 1000 equal Parts by perpendicular Chords: Take, out of a Table of natural verſed Sines, the Degrees, Minutes, and (by proportioning) the Seconds, anſwering to the verſed Sine propoſed, this gives half the Angle at the Center, or half the Arch of the Segment; then find the Meaſure thereof in Parts of the Radius (1); by multiplying the Number of Seconds therein, by the conſtant Factor .000004848I, expreſſing the Length of one Second; being = $\frac{6.28318}{1296000}$, *viz.* the whole Periphery of the Circle (to the Rad. 1) divided by the Number of Seconds in 360°.

From the Meaſure of the Arc, thus obtained, take $\frac{1}{2}$ the Sine of twice that Arc, the Remainder will expreſs the Meaſure of the Segment to the verſed Sine propoſed, when the Diameter of the Circle is ſuppoſed = 2: But if the Diameter of the Circle be = 1, which indeed is more commodious for Practice; then, by the preceding Theorem, the *Meaſure* of any Segment will only be $\frac{1}{4}$th of *that* of a ſimilar Segment, when the Diameter of its Circle is ſuppoſed = 2. By this Means we derive the very ſame Table, as that given at the End of *Shirtcliffe's* Gauging.

Suppoſe, for Example, it was propoſed to find the Meaſure of the Segment of a Circle whoſe Diameter is 2, and the verſed Sine .1.—Let the Radius (1) be conceived to be divided into 1000 equal Parts, then the propoſed verſed Sine will be repreſented by 100; for, by the preceding *Theorem*, 1 : .1 :: 1000 : 100; then, in *Sherwin's* Tab. of nat. verſed Sines, againſt 999.346 we have 25° 50′, and alſo againſt 1000.614 we have 25° 51′; ∴ 1.268 (*viz.* 1000.614—999.346) : 60″ :: .654 (1000—999.346) : $\frac{.654 \times 60}{1.268}$ = 31″ *very nearly*; then will 25° 50′ 31″ expreſs half the Arc of the Sector (or Segment), the double whereof is 51° 41′ 2″, the Sine of which is .7845961; but 25° 50′ 31″ = 93031″, which being multiplied by .000004848I, Sine and Diameter of a Circle are known, (or the verſed Sine and the Chord of a Segment of a Circle from whence the Diameter becomes known, ſee *Page* 88); then will the Meaſure of the Segment be obtained by the following eaſy

RULE.

Divide the verſed Sine of the Segment (with a competent Number of Cyphers annexed) by the Diameter of its Circle, to three Places of Decimals in the Quotient; find this *Quotient* in the Table of Areas of the Segment of a Circle, under the Letters V. S, and then againſt it, under *Seg. Area*, is a Decimal Number; which being multiplied by the Square of the given Diameter, the Product will be the required Meaſure of the Segment.

EXAMPLE.

Suppoſe the Diameter of a Circle to be 80 Inches; required the Area of a Segment thereof (in Ale Gallons, *&c.*) whoſe verſed Sine is 30 Inches.

OPERATION.

.0000048481, the Length of the Arc of 1″ (to the Radius 1), gives .45102359, for the Meaſure of the [Arc 25° 30′ 31″ in Parts [of the Radius 1.

Subtract ½ the Sine of the whole Arc, *viz.* Half .7845961 . . . } .39229805

Remains the required Area, when the Diameter is = 2 . . . . . } .05872554

¼th of which (ſee the preceding *Theo.*) is .01468138, the Area of the Segment of a Circle whoſe Diameter is 1, and verſed Sine .1 or .05; *viz.* 100 or 50, according as the Radius is ſuppoſed to be divided into 1000, or 500 equal Parts.

OPERATION.

```
80) 30.000 (.375 Quotient.
    240
    ———
     600
     560
     ———
      400
      400
      ———
       . .
```

Under the Letters V. S, find the above Quotient .375, againſt which is .269013, this being multiplied by 6400, the Square of the Diameter, the Product is 1721.6832, the Area of the Segment in Inches; which being divided by 282 gives 6.105 Ale Gallons, and being divided by 231, the Quotient will be 7.45 Wine Gallons.

If the Area of the above Segment be computed by the foregoing *general Rule*, the Reſult will be 6.105 Ale, and 7.45 Wine Gallons, *exactly as above.*

PROP. XI.

The tranſverſe and conjugate Diameters of an Ellipſis being given; to determine the Area thereof, in Ale and Wine Gallons, and Malt Buſhels.

RULE.

Multiply the tranſverſe (or longeſt) Diameter, by the conjugate (or ſhorteſt) Diameter; and divide the Product by 359 for Ale, 294 for Wine Gallons, and 2738 for Malt Buſhels.

EXAMPLE.

Suppoſe the tranſverſe Diameter of an Ellipſis to be 70, and the Conjugate 50 Inches; required its Area in Ale Gallons, *&c.*

OPERATION.

$$\begin{array}{r}70\\ 50\\ \hline\end{array}$$

359)3500.00(9.75, the Area in A. Gallons, *nearly.*
294)3500.00(11.90 Wine Gallons.
2738)3500.00(1.27 Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{matrix}359\\294\\2738\end{matrix}\right\}$ on A, ſet 50 on B; then againſt 70 on A, we ſhall have $\left\{\begin{matrix}9.75\\11.90\\1.27\end{matrix}\right\}$ on the Line B, the Areas *as before.*

PROP. XII.

Having given the Baſe and Perpendicular of a Parabola, (*or the Ordinate and Abſciſſa,* ſee Defin. 9, Page 63); *to determine the Area thereof, in Ale and Wine Gallons, and Malt Buſhels.*

RULE.

Multiply the Baſe (or Ordinate) by $\frac{2}{3}$ds of the Perpendicular (or Abſciſſa); and divide the Product by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

EXAMPLE.

EXAMPLE.

Let the Baſe (or Ordinate) of a Parabola be 64, and the Perpendicular (or Abſciſſa) 36 Inches; required the Area in Ale Gallons, *&c.*

OPERATION.

64
⅔ds of 36 (the Perp.) is 24

256
128

282)1536(5.44, the Area in Ale Gallons.
231)1536.00(6.64 Wine Gallons.
2150)1536.00(0.71 Malt Buſhel.

By the Sliding-Rule.

To $\left\{\begin{matrix}282\\231\\2150\end{matrix}\right\}$ on A, ſet 64 (or 24) on B; then oppoſite 24 (or 64) on A, we ſhall have $\left\{\begin{matrix}5.44\\6.64\\0.71\end{matrix}\right\}$ on the Line B, the above Areas.

PROP. XIII.

Having the tranſverſe and conjugate Diameters of an Ellipſis given; to find the Area of any Segment thereof, (formed by drawing a Line parallel to either of thoſe Diameters.)

RULE.

Find (by *Prop.* 10. *Page* 89.) the Area of a circular Segment, whoſe verſed Sine is the Altitude of the elliptic Segment, and the Diameter of the Circle is the tranſverſe (or conjugate) Diameter of the Ellipſis: Then, if the elliptic Segment is formed by a Line parallel to the conjugate Diameter, multiply the Area of the circular Segment by the Conjugate, and divide the Product by the tranſverſe Diameter: But if the elliptic Segment is made by a Line drawn parallel to the tranſverſe Diameter; then multiply the Area of the ſaid circular Segment by the tranſverſe Diameter, and divide the Product by the Conjugate,[*] the Quotient (in each Caſe) will be the Area of the elliptic Segment (in Inches, *&c.*); which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

EXAMPLE.

---

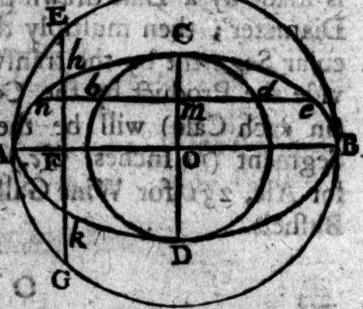
[*] Let the tranſverſe Axis AB $=a$, the Conjugate CD $=c$, the Abſciſſa C$m=x$, and the Ordinate $mn=y$ (ſee the following *Fig.*): Then, for the very ſame Reaſon that $\frac{c}{a}\times\dot{x}\sqrt{ax-x^2}$ (or $y\dot{x}$) is the Fluxion of the elliptic Segment *kAb* (when AF $=x$ and F$b=y$), will $\frac{a}{c}\times\dot{x}\sqrt{cx-x^2}$ be the Fluxion of the elliptic Segment *n*C*e*; but $\dot{x}\sqrt{cx-x^2}$ is the Fluxion of the circular Segment *b*C*d* (ſee *Simpſon*'s Fluxions, Page 146); let the Fluent thereof be $=$ A; then the Fluent of $\frac{a}{c}\times\dot{x}\sqrt{cx-x^2}$, is $\frac{a}{c}\times$ A; whence the Area of the *circular Segment b*C*d*, is to that of the *elliptic Segment n*C*d*; as A : $\frac{a}{c}\times$ A (or 1 : $\frac{a}{c}$) or $c : a$; that is, as CD : AB :: *b*C*d* : *n*C*e*.

EXAMPLE.

In the Ellipſis ADBC, let the tranſverſe Diameter AB be 82, and the Conjugate CD be 52 Inches, and ſuppoſe the verſed Sine AF to be 8.5 Inches; to find the Area of the elliptic Segment *kAb*, in Ale Gallons, *&c.*

OPERATION.

82) 8.500 (.103



Againſt the verſed Sine .103, in the Table for the Segments of Circles, is .042687, which multiply by the Square of 82, *viz.* 6724, gives 287.027, the Area (in Inches) of the circular Segment GAE; this Area being multiplied by 52, and the Product thereof divided by 82 (agreeable to the former Part of the preceding Rule) the Quotient will come out 182.01 Inches, the Area of the elliptic Segment *kAb*: Whence the Area of the ſaid Segment, in Ale and Wine Gallons, and Malt Buſhels, is eaſily found; by dividing 182.01 by the above Diviſors reſpectively.

The Method of Operation, for finding the Area of the elliptic Segment *nCe*, is the ſame as above; only obſerve, that the Area of the circular Segment (*bCd*) is to be multiplied by the tranſverſe (inſtead of the conjugate) Diameter, and that Product divided by the conjugate (inſtead of the tranſverſe) Diameter.

PROP. XIV.

To find the Side of a Square inſcribed in a Circle, whoſe Diameter is given.

RULE.

Multiply the given Diameter of the Circle by .707, and the Product will be the Side of the required Square, *nearly.**

EXAMPLE.

Suppoſe the Diameter of a Circle be 62.5 Inches; what is the Side of the inſcribed Square; or the *greateſt* that can be formed within that Circle?

OPERATION.

62.5
.707
———
4375
43750
———

Product is 44.1875, the Side of the Square.

By the Sliding-Rule.

To Unity on A, ſet .707 (marked *s. i*) on B; then againſt 62.5 on A, is 44.2 on B.

---

* In every Circle, the Chord of 90° is manifeſtly the Side of the inſcribed Square; and therefore, when the Diameter of the Circle is Unity, the Side of its inſcribed Square will (by 47. *Eu.* 1.) be expreſſed by $\sqrt{\frac{1}{2}}$, or .707 *&c.* whence, by ſimilar Triangles, it will be, as 1 : .707 :: *a* (any given Diameter) : *a* × .707, the Side of the Square inſcribed in a Circle, whoſe Diameter is *a*, *nearly*. Q. E. I.

N. B. This Propoſition is very uſeful in the quartering of a round Tun, *&c.* as will be ſhewn farther on.

It may be proper to obſerve, that when the Content of any Veſſel is known in cubic Inches, its Content in Pounds of Glaſs may readily be obtained, by dividing the ſaid cubic Inches by the proper Diviſor, as follows.

			Diviſors.	
A Pound Avoirdupoize Weight of	*Flint Glaſs*	contains	8.46	cubic Inches.
	Plate Glaſs		9.178	
	Crown and *Broad Glaſs*		10.516	
	Phial and *Bottle Glaſs*		10.178	

Vid. the *Officer's Inſtructions* for charging the Duties on Glaſs.

Hence the correſponding circular Diviſors, Factors, and Gauge-points, may eaſily be obtained, by the Methods laid down *Pa.* 82.

SECTION VIII.

Of the Measure of solid Figures; *or of finding their Contents in Ale and Wine Gallons, and Malt Buſhels.*

THE Meaſure of every ſolid Figure is computed from another Solid, of a determinate Form and Magnitude; namely, from a *Cube*, whoſe Side is one *Inch*, *Foot*, *Yard*, *&c.* called the *meaſuring Unit*; and the Number of ſuch *Cubes*, or *Units*, (and Parts of an Unit) that any Solid is found to contain, is called the *Meaſure*, or *Content*, of the Solid; therefore when the Meaſure of any ſolid Figure in cubic Inches is known, its Meaſure in Ale and Wine Gallons, and Malt Buſhels will be eaſily found, by dividing the ſaid cubic Inches by the proper Diviſors for thoſe Meaſures reſpectively.

Prop. I.

The Side of a Cube being given in Inches; to find its Content in Ale and Wine Gallons, and Malt Buſhels.

Rule.

The Length, Breadth, and Altitude of the Cube (which are all equal), being multiplied together gives the Content in cubic Inches; which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

EXAMPLE.

Suppoſe the Side of a Cube to be 15 Inches; required its Content in Ale Gallons, *&c.*

OPERATION.

```
    15
    15
  ----
    75
   15
  ----
   225
    15
  ----
  1125
  225
  ----
```

The Content of the Cube in Inches 3375

282)3375.00(11.96 Ale Gallons.
231)3375.00(14.61 Wine Gallons.
2150)3375.00(1.57 Malt Buſhel.

By the Sliding-Rule.

$$\text{To}\left\{\begin{matrix}16.79\\15.19\\46.37\end{matrix}\right\}\text{on D, ſet 15 on C; then oppoſite}$$

$$\text{15 on D, is}\left\{\begin{matrix}11.96\\14.61\\1.57\end{matrix}\right\}\text{on C, the ſame as before.}$$

PROP.

PROP. II.

The Length, Breadth, and Depth (or Altitude) of a rectangular Parallelopipedon being given in Inches; to find its Content in Ale and Wine Gallons, and Malt Buſhels. (See Definition 25, *Pa.* 58.)

RULE.

Multiply the Length by the Breadth, and that Product by the Depth (or Altitude), the laſt Product will be the Content in cubic Inches; which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.

EXAMPLE.

The Length of a rectangular Parallelopipedon is 72, the Breadth 33, and the Depth (or Altitude) 82 Inches; required the Content in Ale and Wine Gallons, and Malt Buſhels.

OPERATION.

Length 72
Breadth 33

216
216

The Area of the Baſe 2376
82

4752
19008

Content in cubic Inches 194832

282)

282)194832.00(690.89 the Content in Ale Gallons.

231)194832.00(843.42 Wine Gallons.

2150)194832.00(90.61 Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{array}{c}282\\231\\2150\end{array}\right\}$ on A, ſet 33 (or 72) on B; then against 72 (or 33) on A, is $\left\{\begin{array}{c}8.4\\10.2\\1.11\end{array}\right\}$ on B.
Areas.

Then to $\left\{\begin{array}{c}8.42\\10.2\\1.11\end{array}\right\}$ on A, ſet 1 on B; and oppoſite 82 on B, is $\left\{\begin{array}{c}690.8\\843.4\\90.6\end{array}\right\}$ on A.

Otherwiſe, by the Sliding-Rule.

A geometrical mean Proportional between the Length (72) and the Breadth (33) is 48.*75* (found by *Prop.* 3. *Page* 46): Then,

To $\left\{\begin{array}{c}16.79\\15.19\\46.37\end{array}\right\}$ on D, ſet 82 on C, and againſt 48.*75* on D, is $\left\{\begin{array}{c}690.89\\843.42\\90.61\end{array}\right\}$ on C, the ſame as above.

The foregoing Methods are both very exact and expeditious, for computing the Contents of many Brewers *Guile-Tuns* and Diſtillers *Waſh-Backs*; that is, ſuch whoſe four Sides ſtand perpendicular to the Bottom, which (in this Caſe) is a rectangular Parallelogram: But, it is to be obſerved, that, on Account of the Unevenneſs of the Sides of the Tun *&c.* it will be very neceſſary to take 10 Lengths (at leaſt), and as many Breadths; each, as near as poſſible, at equal Diſtances from one another, and from the Sides of the Veſſel; then the Sum of the Lengths being divided by 10 (or the Number of Lengths taken) and the Sum of the Breadths divided in like Manner; or, which comes to the ſame Thing, the Decimal Point, in each of theſe Sums (*viz.* when there are 10 Lengths and 10 Breadths), being removed one Place more towards the left Hand, gives the mean Length and Breadth of the Tun, or Back: Suppoſe, in the laſt Example, the Lengths and Breadths were taken, each at 10 different Places, as follow:

	Lengths.	Breadths.
	71.8	32.7
	72.4	33.4
	72.2	33.1
	71.7	33.0
	72.0	32.7
	71.8	32.9
	71.8	33.2
	72.0	33.1
	72.2	33.3
	72.1	32.6
Total	720.0	330.0
$\frac{1}{10}$th is the mean Length	72.00	$\frac{1}{10}$th is 33.00 the mean Breadth.

At common Brewers, *&c.* the Coolers (or Backs) are generally in the Form, as deſcribed above; but, as their Depths ſeldom exceed 8 Inches, it will be ſufficient to take (about the Middle) only one Length, and one Breadth; which being multiplied together, and the Product thereof divided by 282, the Quotient will be the Area of the Cooler in Ale Gallons.

EXAMPLE.

Suppoſe the Length of a Cooler to be 112.5, and the Breadth 82.2 Inches; required its Area in Ale Gallons.

OPERATION.

```
       112.5
        82.2
     -----------
        2250
       2250
      9000
     -----------
282)9247.50(32.8 Area in Ale Gallons.
```

The Worts in the Coolers being always gauged to the tenth of an Inch, it is therefore neceſſary that the Tables of ſuch Veſſels ſhould be made in Barrels, Firkins, *&c.* to every Tenth, which is uſually called *tenthing* a Cooler: But, before we enter upon that, it will be proper to obſerve, that, at common Brewers, there is an Allowance made of one Gallon in ten, on Account of the Heat of the Worts; that is, every 10 Gallons of hot Worts in the Coolers, will be but 9 Gallons when cold, and let down into Tun; conſequently a Table muſt be made of only $\frac{9}{10}$th of the whole Area of the Cooler, as follows.

```
Whole Area      32.8 Gallons.
Subtract 1/10th  3.28
                -----
Remains         29.52, the neat Area for one
```

Inch, $\frac{1}{10}$th of which is 2.952, the *neat Area* of the Cooler for $\frac{1}{10}$th of an Inch; but the 3d Decimal Figure (being in this Caſe of ſmall Value) may be rejected, and therefore 2.95 will be the neat

Area; by the continual Addition of which, the following Table of *Beer Barrels* was made.

Note. The neat Area of any Back (or Cooler), for $\frac{1}{10}$th of an Inch, may alſo be found by multiplying the whole Area thereof (*viz.* for one Inch deep) by .09: — Thus, for Example, the foregoing Area 32.8 being multiplied by .09, gives 2.952, the neat Quantity for $\frac{1}{10}$th, the ſame as before.

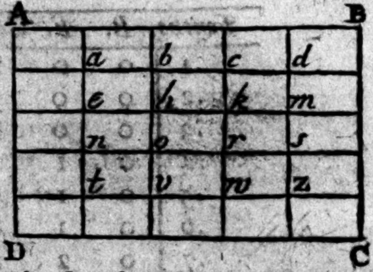
Tenths.	*B.*	*F.*	*Gall.*	*Parts.*
.1	0	0	2	95
.2	0	0	5	90
.3	0	0	8	85
.4	0	1	2	80
.5	0	1	5	75
.6	0	1	8	70
.7	0	2	2	65
.8	0	2	5	60
.9	0	2	8	55
1.0	0	3	2	50
&c.				

I thought it needleſs to proceed any farther with the foregoing Table, ſeeing that the Method of forming it, is only the continual Addition of 2.95 Gallons.

By the ſame Method the Back may be tabulated for *Ale Barrels, &c.* (*i. e.* 34 Gallons to a Barrel), due Regard being had to the Decimal Parts, when the Sum of the Gallons and Parts, of the two Numbers to be added, exceeds one Firkin (*i. e* 8.5 Gallons).

It is, Indeed, wholly immaterial in what Part of a Cooler the Gauge of the Worts is taken, provided its *Bottom* is fixed in an horizontal Poſition: But it is well known, that *that* is always placed a little inclined, for the Convenience of the Wort's running out: Beſides, at common Brewers, large Backs are generally found to ſettle of themſelves, more one Way than another; and moreover their Bottoms will frequently warp, and thereby cauſe ſuch an Unevenneſs in them, as to render it almoſt impoſſible to know where to take a Dip of the Worts, whereby their true Quantity may be aſcertained.

Now in order to find, with the moſt Certainty, a mean Dip of a Back, or Cooler, proceed thus: Let its Length and Breadth be each divided at the Bottom, into 4, 5, 6, 7, *&c.* equal Parts, according to the Magnitude of the Back, and the Irregularity of its Bottom, alſo let parallel Chalk-lines be ſtruck; ſee the above Figure ABCD, which may be ſuppoſed to repreſent the Bottom of a rectangular Cooler: Then (the Bottom being covered with Water) let Dips be taken at all the Points of Interſection (*a*, *e*, *n*, *t*, *v*, *&c.*) of thoſe parallel Lines, the Sum of which Dips being divided by the Number of Dips taken, will give the mean Depth (or Dip) ſought.



Find in what *Place* of the Back, a Dip being taken, will anſwer to the mean Dip, for *that* muſt be *noted* for the conſtant Dipping-place: But if ſuch Place cannot be eaſily come at, then chooſe One which will be the moſt convenient to dip at, and there make ſome immoveable Mark; obſerve how much the Dip taken at this Place falls ſhort, or exceeds the mean Dip (found as above), and accordingly mark it down on the Side of the Back, at the fixed Dipping-place, with the Character + or —: Suppoſe, for Example, the mean Dip of a Cooler be 4.5; and, at the intended Dipping-place, it is found to dip only 4 Inches; therefore it is plain that .5 muſt be added to every Gauge (or Dip) that is taken of the Worts, at the fixed Dipping-place, and muſt be there marked thus, + 0.5: But if the Dip, at this Place, had been 5 Inches, (which exceeds the mean Dip by .5), we muſt then have marked the Dipping-place — 0.5.

Note. If the Sides of a Back *&c.* are parallel, and there happens to be any conſiderable Difference between the two Diagonals: — Then we muſt mark (with a Chalk-line on the Bottom) the longeſt Diagonal, and let Perpendiculars fall thereon, from the two oppoſite Angles, as in the Trapezium *Pa.* 74.

It will be unneceſſary to give Examples for finding the Contents of all the various Sorts of Priſms which may occur in Practice, if the 25th *Definition*, *Pa.* 58, be rightly underſtood; for the Method of Operation, by the Pen, is much the ſame as *that* of the foregoing Examples, let the Figure of the two equal Ends of the Priſm be what it will: — That is, *multiply the Area of one of the Ends, by their perpendicular Diſtance aſunder, and the Product will be the Meaſure of the Priſm in cubic Inches; which divide by 282 for Ale, 231 for Wine Gallons, and 2150 for Malt Buſhels.*

PROP. III.

The Diameter and Length of a Cylinder being given; to find its Content in Ale and Wine Gallons, and Malt Buſhels.

RULE.

Multiply the Square of the Diameter by the Length (or Altitude) of the Cylinder, and divide the Product by 359 for Ale, 294 for Wine Gallons, and 2738 for Malt Buſhels: — Or the ſaid Product being multiplied (ſee *Ta. Pa.* 83) by .0027851,

.0027851, .0034 and .000365, will give the Content of the Cylinder in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE.

The Diameter of a Cylinder BC is 32, and the Altitude (or Length) AB 45.5 Inches; required its Content in Ale Gallons, *&c.*

OPERATION.

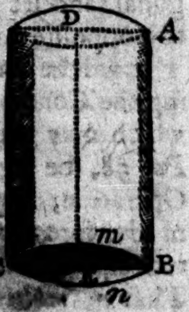
32
32

64
96

The Square of the Diam. 1024
Length 45.5

5120
5120
4096

359)46592.00(129.78 the Content in Ale Gallons.
294)46592.00(158.47 Wine Gallons.
2738)46592.00(17.01 Malt Buſhels.



By the Sliding-Rule.

To $\left\{\begin{array}{l}18.95\\17.15\\52.32\end{array}\right\}$ marked $\left\{\begin{array}{l}\text{A.G}\\\text{W.G}\\\text{M.R}\end{array}\right\}$ on D, ſet 45.5 on C; then againſt 32 on D, is $\left\{\begin{array}{l}129.78\\158.47\\17\end{array}\right\}$ on C, the ſame as before.

If the Diameter be leſs than 10, or more than 100; or if it ſo happens, that, when the Length of the Cylinder on C is ſet to any of the foreſaid Gauge-points on D, the Diameter of the Cylinder on D, ſhould fall off the Slide either towards the right or left Hand: Then, in order to find the Content of the Cylinder by the *Sliding-Rule*, we may have Recourſe to the Method laid down at *Pa.* 48.

Thus, for Example; ſuppoſe the Length of a Cylinder be 45.5, and the Diameter thereof 8 Inches; required its Content in Ale and Wine Gallons, and Malt Buſhels.

Now it is very evident, from the Conſtruction of the Rule, and the Gauge-points thereon, that the Diameter of the Cylinder (8) will fall off the Line D: But,

If to $\left\{\begin{array}{l}18.95\\17.15\\52.32\end{array}\right\}$ on D, be ſet 45.5 on C; then againſt 16 (the Double of 8) on D, we ſhall have $\left\{\begin{array}{l}32.5\\39.6\\4.25\end{array}\right\}$ on C; which being divided by 4 (becauſe the Diameter of the Cylinder was doubled), gives $\left\{\begin{array}{l}8.12\text{ Ale Gallons}\\9.9\text{ Wine Gallons}\\1.06\text{ Malt Buſhels}\end{array}\right\}$ the required Content of the Cylinder.

PROP. IV.

To find the Content of a Pyramid (or Cone), in Ale and Wine Gallons, and Malt Buſhels. (See Definitions 1 and 27, *Pages* 59 and 61.)

RULE.

RULE.

Multiply the Area of the Baſe (let the Figure thereof be what it will) by $\frac{1}{3}$d of the Altitude, and the Product will be the Content in cubic Inches; which being divided by 282, 231, and 2150, the Quotient will be the required Content in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE.

Suppoſe the Side of the Baſe of a ſquare Pyramid to be 35, and the Altitude 57 Inches; required its Content in Ale Gallons, *&c.*

OPERATION.

35
35

175
105

The Area of the Baſe in Inches 1225
$\frac{1}{3}$d of the Altitude 19

11025
1225

The Content of the Pyramid in Inches 23275

282)23275.00(82.53 the Content in Ale Gallons.

231)23275.00(100.75 Wine Gallons.
2150)23275.00(10.82 Malt Buſhels.

By the Sliding-Rule.

To $\left\{\begin{matrix}16.79\\15.19\\46.37\end{matrix}\right\}$ on D, ſet 19 on C; then againſt 35 on D, is $\left\{\begin{matrix}82.53\\100.75\\10.82\end{matrix}\right\}$ on C, the ſame as before.

PROP. V.

To find the Content of the Fruſtum of a Cone (or Pyramid of any Kind) in Ale and Wine Gallons, and Malt Buſhels. (See Defin. 32. *Pa.* 60.)

RULE.

To the Sum of the Areas of the two Ends of the Fruſtum, add a Geometrical-Mean between thoſe Areas (*viz.* the Square Root of their Product), multiply this Sum by $\frac{1}{3}$d of the Altitude of the Fruſtum, the Product will give the Content thereof in cubic Inches;* which being divided as in the laſt Example, gives the Content ſought.

The

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*Let the *Meaſure* of the greater End of the Fruſtum of any Cone, or Pyramid whatever (ſee *Fig.* V.), be denoted by A^2, *that* of the leſſer End by a^2; then the Diameters, or any two homologous Sides of thoſe Ends (becauſe of their Similarity), will be as A and *a* reſpectively: Moreover, let the Diſtance (*bm*) of the Ends be denoted by p, and the Perpendicular $Fn = x$; and let the Solid (*Aacb*BA) be cut by a Plane parallel to one of its Sides, and ſo as to form a Pyramid (or a Priſmoid) HI*b*B, and parallel to that Plane, let another be ſuppoſed to paſs from any Altitude x: Then it is manifeſt, that p (*bm*) : $A-a$ (which is as BH) :: x (*Fn*) : $\frac{A-a}{p} \times x$ (which is as B*r*); ∴ $A - \frac{Ax-ax}{p}$ (being as AB−B*r*) will be as EF; conſequently

The preceding Rule is general, let the Figure of the two (ſimilar) Ends of the Fruſtum be what it will; but the Content of the Fruſtum of a Cone in Ale Gallons, *&c.* is more expeditiouſly obtained by the following

RULE.

From the Square of the Sum of the top and bottom Diameters, ſubtract the Product of thoſe Diameters; the Remainder being multiplied by the Altitude of the Fruſtum, and the Product divided by 1077 for Ale, 882.36 for Wine, and 8214 for Malt Buſhels, gives the required Content.†

EXAMPLE.

---

conſequently the Area of the Section E G F (parallel to the Ends of the Solid) will be expreſſed by $\overline{\frac{Ap-Ax+ax}{p}}\Big|^2$; whence the Fluxion of the Solid, univerſally, will be $\overline{\frac{Ap-Ax+ax}{p}}\Big|^2 \times \dot{x}$, or (when expanded) $\frac{A^2p^2\dot{x}-2A^2px\dot{x}+2Apax\dot{x}+A^2x^2\dot{x}-2Aax^2\dot{x}+a^2x^2\dot{x}}{p^2}$, whoſe Fluent is $\frac{A^2p^2x-A^2px^2+Apax^2+\frac{A^2x^3}{3}-\frac{2Aax^3}{3}+\frac{a^2x^3}{3}}{p^2}$; which, when $x=p$, becomes $Apa+\frac{A^2p}{3}-\frac{2Apa}{3}+\frac{pa^2}{3}$, or $\overline{A^2+Aa+a^2} \times \frac{p}{3}$. Q. E. I.

† Suppoſe B and *b* denote the Diameters of two Circles, whoſe Areas are A^2 and a^2 reſpectively; that is, let $B^2 \times .7854 = A^2$ and $b^2 \times .7854 = a^2$; then will $Bb \times .7854 = Aa$, and $\therefore \overline{B^2+Bb+b^2} \times .7854 = A^2+Aa+a^2$; whence $\overline{B^2+Bb+b^2} \times .7854 \times \frac{p}{3}$ $(= \overline{\overline{B+b}|^2 - Bb} \times .7854 \times \frac{p}{3})$ $= \overline{A^2+Aa+a^2} \times \frac{p}{3} =$ the Content of a Fruſtum of any pyramidic Solid whoſe Altitude is p; ſee the preceding *Note.*

Hence

EXAMPLE.

Let the top Diameter be 22, the bottom Diameter 40, and the Altitude (or Depth) 60 Inches; required the Content of the Fruſtum in Ale Gallons, &c.

OPERATION.

22
40

62 22
62 40

124 880, the Product of the Diameters.
372

Sq. of the Sum of the Diam. } 3844
Subtract 880

Remainder 2964
Depth 60

1077)177840(165.12, the Content of the Fruſtum in Ale Gallons; whence, by the proper Diviſors, the Content in Wine Gallons and Malt Buſhels, will be found to be 201.55 and 21.63 reſpectively.

Hence it is very eaſy to deduce another general Rule for determining the Content of the Fruſtum of a Cone: For $\overline{B-b}|^2 + 3Bb \times .7854 \times \frac{p}{3}$ ($= \overline{B^2+Bb+b^2} \times .7854 \times \frac{p}{3}$) is likewiſe $= \overline{A^2+Aa+a^2} \times \frac{p}{3}$.

Provided a Brewer's Guile-Tun (or a Diſtiller's Waſh-Back, *&c.*) was a perfect Fruſtum of a Cone, the above Rule would be ſufficient for finding its Content; and moreover a general Method might be given for finding the *true* Quantity upon every Inch of the Fruſtum's Altitude, by which Means a Table might be formed to know the Quantity of Liquor contained in the Tun (or Back), at any Number of wet (or dry) Inches of its whole Depth.

But, in Veſſels of this Kind, it is well known that the Croſs-Diameters differ pretty much in various Parts of the Altitude, eſpecially if the Veſſel is of a conſiderable Magnitude: Therefore the moſt practical and certain Method of finding the Content, and tabulating a Guile-Tun, *&c.* reſembling the Fruſtum of a Cone is, to take Croſs-Diameters at the Middle of every 6, 7, 8, 9, or 10, *&c.* Inches of its Altitude; then will half the Sum of any two Croſs-Diameters be, *nearly*, the true Diameter at that particular Altitude.

Find the Areas in Ale or Wine Gallons correſponding to the Diameters, thus obtained, in the Middle of every 6, 7, 8, 9, or 10, *&c.* Inches; the Sum of theſe Areas being multiplied by their common Diſtance aſunder, or, which is the ſame Thing, each Area being multiplied by its correſponding Part of the Depth, the Sum of the Products will give the Content of the Tun in Gallons, if it ſtands perpendicular to the Horizon: But if the Tun ſtands inclined; then ſo much Liquor muſt be meaſured therein, as will be ſufficient to cover the Bottom, and at the Place where the Depth of the Tun was found (which is always taken at the intended Dipping-place), take the Depth of the Liquor which covers the Bottom;

that being ſubtracted from the whole Depth, leaves the *neat Depth* of the Tun.

Now, to quarter ſuch a Tun, or to obtain Croſs Diameters of any Veſſel of a circular Form, in any Part of its Altitude, proceed thus.

With a Chalk-Line and Plummet, at the loweſt Part of the Tun, ſtrike a ſtrait Line on the Side thereof, from the Bottom to the Top; then with a Dimenſion-Rod take the Diameter of the Tun at the Bottom, multiply that Diameter by .7 (ſee *Pa.* 99) the Product will be the Side of the inſcribed Square, *very nearly*, with which the Tun may be quartered at the Bottom; and by the very ſame Method of proceeding, the Tun (or Back) may be quartered at the Top; then let Marks be made with Chalk at each Quarter, both at the Bottom and Top, and ſtrike ſtrait Chalk-Lines from thoſe Marks; the Veſſel will then be properly quartered, and Croſs-Diameters may be taken at any aſſigned Diſtance from the Bottom.

It may be proper to obſerve, that by Means of quartering a round Tun (or any inclined circular Veſſel) both at the Top and Bottom, we get the *true* Croſs-Diameters at any Part of its Depth; which otherwiſe could not be obtained, unleſs the Veſſel was fixed perfectly upright, or with its Bottom parallel to the Horizon.

It is moreover to be obſerved, that, when great Exactneſs is required, the Number of Areas, in every Veſſel (whether ſtrait or curve-ſided) ſhould be ſuch, that the Increaſe of the Croſs-Diameters (or Dimenſions) may not exceed one Inch: — In Order to obtain which, in a ſtrait-ſided Veſſel; divide the Altitude thereof by the Difference of the top and bottom Diameters (or Dimenſions), and the Quotient will be the perpendicular Diſtance which the Croſs-Diameters, &c. are to be taken from each other.*

If a Veſſel is to be tabulated for the dry Inches, it will be proper to begin from the *Top*, to mark out and take its Dimenſions; and from the *Bottom*, when it is to be tabulated for the wet Inches.

When the Difference of the top and bottom Diameters (or Dimenſions) of any ſtrait-ſided Veſſel is but ſmall; then the Diſtance of the Croſs-Diameters, &c. may be ſet off upon the Side, without ſenſible Error: But, when that Difference is large, we muſt, in Order to have the *true* Diſtance of the Croſs-Diameters &c. on the Side, take the following Method: — Meaſure the ſlant Side of the Veſſel, and multiply the Length thereof by the intended Diſtance of the Croſs-Diameters (or Dimenſions), and divide the Product by the perpendicular Depth of the Veſſel; and the Quotient will be the Diſtance of the Croſs-Diameters, &c. meaſured on the ſlant Side.

Note. It is both more *expeditious* and *certain*, to mark the Sides of any Veſſel, where the Dimenſions are to be taken, with a Pair of *Compaſſes* (ſuch as are uſed by *Coopers*), than by any other Method that has yet occurred to me.

EXAMPLE.

Let the Depth of a Diſtiller's round Waſh-Back be 61.8 Inches, the Drip, or Depth of the Liquor, at the intended Dipping-place 1.2, and the

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* If the Altitude of any ſtrait-ſided Veſſel be denoted by a, and the Difference of the top and bottom Dimenſions by d; then it is evident (by ſimilar Triangles) that $d : a :: 1$ (*viz.* one Inch) : $\frac{a}{d}$ = the perpendicular Diſtance of the Dimenſions.

Croſs-Diameters as below; required the Content of this Veſſel in Wine Gallons.

Inches.		*Diam.*	*Diam.*	*Areas.*	*Gallons.*
12	(6 Inches fr. the Top)	52.8	51.6	9.26	111.12
10	(17 from the Top)	53.5	52.5	9.55	95.50
10	(27 from the Top)	54.3	53.5	9.87	98.70
10	(37 from the Top)	55.0	54.4	10.17	101.70
10	(47 from the Top)	55.9	55.4	10.56	105.60
8.6	(56.3 fr. the Top)	56.6	56.3	10.83	93.13
Drip 1.2					10.00
Depth 61.8				Content	615.75 Wine Gallons.

		Gal.
Groſs Depth	61.8	
Drip	1.2	10
Neat Depth	60.6	

The Manner of finding the foregoing Areas, *&c.* is extremely eaſy: Thus, for Inſtance, the Sum of the two Croſs-Diameters at the Top being 104.4, the Half of which is 52.2, the Diameter at 6 Inches from the Top of the Veſſel; then againſt this Diameter, in the Table of Wine Areas, we have 9.26, which being multiplied by 12, gives 111.12 Wine Gallons, the Content for the firſt 12 Inches from the Top of the Back. By the very ſame Method the other five Areas, *&c.* were obtained.

In Order to form the preceding Work into a Table, whereby the Quantity of Liquor in the Back, at any Number of dry Inches, may be known by Inſpection; proceed thus: From the whole Content 615.75, ſubtract 9.26, the Area in the Middle of the firſt 12 Inches from the Top, *viz.* 9.26, and the Remainder 606.49 will ſhew the Quantity in the Back at one Inch dry; again, from 606.49 take 9.26, the Remainder 597.23 will be the Quantity in the Back at two Inches dry; and by proceeding in the ſame Manner for 10 Inches more, we ſhall get the Quantities of Liquor in the Back at 3, 4, 5, 6, *&c.* to 12 Inches dry.

From the Quantity at 12 dry Inches, ſubtract the 2d Area, *viz.* 9.55, and from the Remainder take again the 2d Area, and ſo on, 'till we come to the 22d Inch; then proceed with the 3d, 4th, 5th, and 6th Areas, ſucceſſively, 'till we get the Quantity in the Back at 60 dry Inches; from which Quantity take $\frac{6}{10}$th of the 6th Area, and the Remainder will be 10 Gallons (the Drip) if the Work is right.

Though it may, perhaps, be reckoned more elegant to determine the Meaſure of the Drip, or Fall, of a Tun, *&c.* by Computation, than to cover its Bottom (with Water) by a known Meaſure; yet I cannot but think (becauſe the Inclination of a Tun, *when fixed*, is ſo *very ſmall*) that the latter Method is far more eligible, both with Reſpect to Expedition and Exactneſs, than to make uſe of a Quadrant, or any other mathematical Inſtrument, to determine this *ſmall* Inclination; and afterwards to have the Meaſure of the Drip to find, by a very troubleſome Computation.

OPERATION *for a* TABLE *of dry Inches, in Wine Gallons.*

Inc.	*Gallons.*	*Inc.*	*Contin.*
Full	615.75	5	569.45
1.*A.*	9.26		9.26
1	606.49	6	560.19
	9.26		9.26
2	597.23	7	550.93
	9.26		9.26
3	587.97	8	541.67
	9.26		9.26
4	578.71	9	532.41
	9.26		9.26

Inch	*Contin.*	*Inch*	*Contin.*
10	523.15	26	369.65
	9.26		9.87
11	513.89	27	359 78
	9.26		9.87
12	504.63	28	349.91
2.*A.*	9.55		9.87
13	495.08	29	340.04
	9.55		9.87
14	485.53	30	330.17
	9.55		9.87
15	475.98	31	320.30
	9.55		9.87
16	466.43	32	310.43
	9.55	4.*A.*	10.17
17	456.88	33	300.26
	9.55		10.17
18	447.33	34	290.09
	9.55		10.17
19	437.78	35	280.92
	9.55		10.17
20	428.23	36	269.75
	9.55		10.17
21	418.68	37	259.58
	9.55		10.17
22	409.13	38	249.41
3.*A.*	9.87		10.17
23	399.26	39	239.24
	9.87		10.17
24	389.39	40	229.07
	9.87		10.17
25	379.52	41	218.90
	9.87		10.17

Inch	*Contin.*	*Inch*	*Contin.*	
42	208.73	52	103.13	
5. *A.*	10.56	6. *A.*	10.83	
43	198 17	53	92.30	
	10.56		10.83	
44	187.61	54	81.47	
	10.56		10.83	
45	177.05	55	70.64	
	10.56		10.83	
46	166.49	56	59.81	
	10.56		10.83	
47	155.93	57	48.98	
	10 56		10.83	
48	145.37	58	38.15	
	10.56		10.83	
49	134.81	59	27.32	
	10.56		10.83	
50	124.25	60	16.49	
	10.56		6.49	$=\frac{6}{10}$th of
51	113.69	60.6	10.00	[10.83.
	10.56	61.8	00	

It is to be obſerved, that, in tabulating any Veſſel, there is no Neceſſity for writing down (as above) the Area at every Inch; but only to enter it on a ſmall Piece of Paper, and move it downwards as we ſubtract, or add, according as the Table is to be made for the dry, or wet Inches: And, that we may proceed with more Certainty, it will be neceſſary to examine the Operation, at every different Area, in the following Manner.

Whole

	Gallons.
Whole Content, ſee *Pa.* 120.	615.75
Subtract 12 times the *top Area*	111.12
Remains, at 12 Inches dry,	504.63
Subtract 10 times the 2*d Area*	95.50
At 22 Inches dry	409.13
Subtract 10 times the 3*d Area*	98.70
At 32 Inches dry	310.43
Subtract 10 times the 4*th Area*	101.70
At 42 Inches dry	208.73
Subtract 10 times the 5*th Area*	105.60
At 52 Inches dry	103.13
Subtract 8.6 times the 6*th Area*	93.13
Remains the *Drip*, or *Fall*,	10

Sometimes the Poſition of a Diſtiller's Waſh-Back, *&c.* is ſuch, that it is found neceſſary to fix the Dipping-place thereof, at ſome certain Diſtance above the Top of the Back, which Diſtance is called the *Curb*; and, to avoid unneceſſary Trouble in tabulating the Veſſel, it is always taken a whole Number, in Inches. Suppoſe, in the preceding Example, there had been a Curb of 9 Inches; then, at the Time of taking the Dimenſions of the Back, we ſhould have written down,

Whole Depth	70.8	
Curb	9.0	
Groſs Depth of the Back	61.8	*Gal.*
Drip	1.2 . .	10
Neat Depth	60.6	

Moreover, in tabulating a Veſſel where there is a Curb, inſtead of beginning at *Full* (as in the preceding Table), we muſt begin as follows:

	Inches.	*Gallons.*
Curb	9 . .	615.75
	10 . .	606.49
	11 . .	597.23
	12 . .	587.97
		&c. as before.

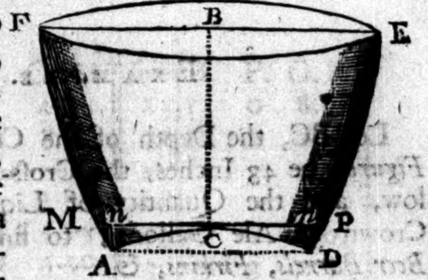
If the foregoing Dimenſions were thoſe of a Brewer's round Guile-Tun; then the Method of finding its Content, and tabulating the ſame, would differ but little from that above exhibited.

For if from the whole Content, found in Barrels, Firkins, and Gallons (and to two Decimal Places of a Gallon), we ſubtract the top Area, reduced in like Manner; the Remainder will ſhew the Quantity, in Barrels, Firkins, *&c.* in the Tun at one Inch dry: Proceed to find the Quantities at the 2d, 3d, 4th, *&c.* dry Inches, and likewiſe with the 2d, 3d, and 4th, *&c.* Areas. — This Method will, I apprehend, be ſufficiently illuſtrated by the following Operation.

To take the Dimenſions of a Copper *with a riſing Crown; to find its Content and tabulate the ſame, in Barrels, Firkins,* &c.

It is well known that Coppers and Stills, are always fixed with their Bottoms ſomewhat inclined to the Horizon, their loweſt Part being at the Cock, for more Convenience of draining off the Liquor; but this Inclination being ſo very ſmall, that the Figure of the Surface of the Liquor, at every Altitude, may be conſidered as a Circle without any, ſenſible, Error reſulting therefrom,

therefrom; therefore the Dimenſions may be taken in the following Manner.



Suppoſe the Figure ACDEF to repreſent a Copper, when fixed, and A the Place of the Cock; through C, the Center of the Crown, extend a Piece of Pack-thread in ſuch a Manner, that the perpendicular Diſtances A*n* and D*n* may be equal to each other, and let Marks be made, on the Sides of the Copper, at M and P; alſo extend a ſmall Cord (or Pack-thread EF) diametrically over the Top of the Copper, and with one End of the Dimenſion-Cane on the Center C, find the neareſt Diſtance to the ſaid Pack-thread EF; that Diſtance (*viz.* BC) will be the internal Altitude of the Copper.

Now let the Copper be quartered, at the Bottom and Top, by the Method already laid down at *Page* 117, for a round Back (or Tun), and let Croſs-Diameters be taken in the Middle of every 6, 7, 8, 9, 10, &c. Inches of the Altitude BC, beginning from the Top; that is, let the firſt Croſs-Diameters be taken, either at 3, 3.5, 4, 4.5, or 5, &c. Inches from EF, and the ſecond Croſs-Diameters be taken either at 9, 10.5, 12, 13.5, or 15 from the Top, and ſo on towards C; then find, by the Table, the Areas in Ale Gallons, correſponding to thoſe Croſs-Diameters, in the ſame Manner as is laid down for Wine Areas, *Pa.* 119; theſe Areas being each multiplied by their correſponding Parts of the Depth, the Sum of the

Products, together with the Quantity which *exactly* covers the Crown ACD, will be the whole Content of the Copper ACDEF.

EXAMPLE.

Let BC, the Depth of the Copper (ſee the laſt *Figure*) be 43 Inches, the Croſs-Diameters as below, and the Quantity of Liquor to cover the Crown 38 Ale Gallons; to find its Content in *Beer Barrels*, *Firkins*, *&c.*

Inches.		Croſs-Di.		*Half the Sum of the* Croſs-Diameters :
13	(6.5 from the Top)	97.5	98.1	97.8
10	(18 from the Top)	95.8	96.4	96.1
10	(28 from the Top)	94.3	93.9	94.1
10	(38 from the Top)	93.2	93.0	93.1

Therefore, by the Table of Ale (or Beer) Areas, *&c.* the Work will ſtand as follows.

		Areas in	*Contents* in	*Areas* in			*Contents* in		
Inches.	*Diam.*	Gal.	Gal.	B.	F.	G.Pts.	B.	F.	G.Pts.
13	97.8	26.63	346.19	0	2	8.63	9	2	4.19
10	96.1	25.72	257.20	0	2	7.72	7	0	5.20
10	94.1	24.66	246.60	0	2	6.66	6	3	3.60
10	93.1	24.13	241.30	0	2	6.13	6	2	7.30
	To cover the Crown		38				1	0	2
Dep. 43	The whole Content		1129.29				31	1	4.29

A TABLE *of the preceding Work: The Method of forming of which has already been obſerved, at* Pa. 124.

Inch.	B.	F.	G.Pts.
Full	31	1	4.29
	1.Ar.2		8.63
1	30	2	4.66
2	29	3	5.03
3	29	0	5.40
4	28	1	5.77
5	27	2	6.14
6	26	3	6.51
7	26	0	6.88
8	25	1	7.25
9	24	2	7.62
10	23	3	7.99
11	23	0	8.36
12	22	1	8.73
13	21	3	0.10
	2.Ar.2		7.72
14	21	0	1.38
15	20	1	2.66
16	19	2	3.94
17	18	3	5.22
18	18	0	6.50
19	17	1	7.78
20	16	3	0.06
21	16	0	1.34
22	15	1	2.62
23	14	2	3.90
	3.Ar.2		6.66
24	13	3	6.24
25	13	0	8.58
26	12	2	1.92
27	11	3	4.26
28	11	0	6.60
29	10	1	8.94
30	9	3	2.28
31	9	0	4.62
32	8	1	6.96
33	7	3	0.30
	4.Ar.2		6.13
34	7	0	3.17
35	6	1	6.04
36	5	2	8.91
37	5	0	2.78
38	4	1	5.65
39	3	2	8.52
40	3	0	2.39
41	2	1	5.26
42	1	2	8.13
43	1	0	2.00
	Remains to cover the Crown.		

PROP. VI.

To find the Content in Ale and Wine Gallons, and Malt Buſhels, of a Veſſel (called a Priſmoid) *whoſe parallel Ends are any diſſimilar Rectangles, and the Sides of it are four plane Surfaces.*

GENERAL RULE.

To the longeſt (or ſhorteſt) Side of the Rectangle at either End, add that Side at the other End (whether it be the Length or Breadth) which is parallel to it; multiply this Sum by the Sum of the other two parallel Dimenſions *(viz.* one at the Top and the other at the Bottom), and to the Product add the Areas of the two Ends; this Total being multiplied by the Height (or perpendicular Altitude), and the Product thereof divided by 1692 for Ale, 1386 for Wine Gallons, and 12902.5 for Malt Buſhels, gives the Content ſought.*

EXAMPLE.

Suppoſe there is a Tun, whoſe parallel Ends are Rectangles, the Length and Breadth of the Top 36 and 32, the Length and Breadth of the Bottom (being, in this Caſe, reſpectively parallel to *thoſe* above) 48 and 40, and the Height 60 Inches; required the Content of the Tun in Ale Gallons, *&c.*

OPERATION.

---

* There is a very elegant Inveſtigation given in *Simpſon's* Fluxions, *2d Ed.* Pa. 179, for determining the Content of a Priſmoid, when the Sides of the Rectangle at one End, are leſs than the parallel Sides of the *other*; and from the ſame Method of Reaſoning (ſuppoſing *a* and *b* to denote the Length and Breadth of the Rectangle at one End, *c* and *d* the Sides of the Rectangle at the *other*, parallel to *a* and *b* reſpectively), it will appear that the Theorem which that illuſtrious Author has there given is general, let the rectangular Ends be what they will.

OPERATION.

The Length at the greater End	48
The Length at the other End which (in this Caſe) is parallel to *that* above	36
Sum	84
The Sum of the other two parallel Dimenſions, or Breadths (in this Caſe)	72
	168
	588
Product	6048
The Area of the greater End (*i. e.* 48 multiplied by 40) . . .	1920
The Area of the leſſer End (*i. e.* 36 multiplied by 32	1152
Total	9120
Multiplied by the Height	60
	547200

1692)547200.00(323.41 Ale Gallons.
1386)547200.00(394.80 Wine Gallons.
12902.5)547200.00(42.4 Malt Buſhels.

Some Authors have aſſerted, that the ſame Rule which gives the Content of a Priſmoid, will alſo hold good in any ſtraight-ſided Veſſel, whoſe parallel Ends are diſſimilar Ellipſes, and any-how poſited; or if one End is an Ellipſis and the other a Circle: * But it appears that this Aſſertion

---

* Let the Ellipſis ABCD (*Fig.* VI.) repreſent the Baſe of the Solid, and the Circle *dcgf* the Top thereof; alſo let *beap* repreſent a Plane of the Section of the Solid, cut any-where parallel to its Ends: Then, drawing a Diameter

is without proper Foundation, and ſeems to have ariſen wholly from the following Suppoſition; namely,

---

Diameter EF, it is very evident, (becauſe the Sides of the Solid in every Plane conceived to paſs through the Centers of the Ellipſis and Circle are Right-lines) that it will be, as B*e* : *ed* :: E*m* : *mb* (:: *a*C : *ac* :: D*p* : *pg*, &c.) let the Poſition of the Diameter be how it will; or, by Compoſition, B*e*+*ed* : *ed* :: E*m*+*mb* : *mb*, ∴ B*d* : *de* :: E*b* : *mb*. Now let the given Semi-tranſverſe OC$=a$, the Semi-conjugate OB$=b$, the Radius O*b*$=r$, O*b*$=d$, the Abſciſſa *na*$=x$, and the Ordinate *mn*$=y$: Moreover, let B*d* be to *de* (or E*b* to *mb*), in every Poſition of the Diameter EF, as 1 to *n*: Then $\overline{d-x}=$O*n*, and $\sqrt{\overline{d-x}|^2+y^2}=$O*m*; ∴ $\sqrt{\overline{d-x}|^2+y^2}-r=mb$, whence $n:1::\sqrt{\overline{d-x}|^2+y^2}-r:\frac{\sqrt{\overline{d-x}|^2+y^2}-r}{n}=$E*b*, conſequently $\frac{\sqrt{\overline{d-x}|^2+y^2}-r}{n}+r=$OE; then, by ſimilar Triangles, $\sqrt{\overline{d-x}|^2+y^2}$ (O*m*) : $\frac{\sqrt{\overline{d-x}|^2+y^2}-r+rn}{n}$ (OE) :: $d-x$ (O*n*) : $\frac{\overline{d-x}\times\overline{\sqrt{\overline{d-x}|^2+y^2}-r+rn}}{n\sqrt{\overline{d-x}|^2+y^2}}$ $\left(=\frac{\overline{d-x}}{n}-\frac{\overline{r-rn}\times\overline{d-x}}{n\sqrt{\overline{d-x}|^2+y^2}}\right)=$OG; again, by ſimilar Triangles, $\sqrt{\overline{d-x}|^2+y^2}$ (O*m*) : $\frac{\sqrt{\overline{d-x}|^2+y^2}-r+rn}{n}$ (OE) :: y (*mn*) : $\frac{y\sqrt{\overline{d-x}|^2+y^2}-\overline{1-n}\times ry}{n\sqrt{\overline{d-x}|^2+y^2}}$ $\left(=\frac{y}{n}-\frac{\overline{1-n}\times ry}{n\sqrt{\overline{d-x}|^2+y^2}}\right)=$EG; but, by the Property of the Ellipſis, EG² $\left(=\frac{\text{OB}^2}{\text{OA}^2}\times\overline{\text{AO}+\text{OG}}\times\overline{\text{AO}-\text{OG}}\right)=\frac{b^2}{a^2}\times a^2-\overline{\frac{d-x}{n}-\frac{\overline{r-rn}\times\overline{d-x}}{n\sqrt{\overline{d-x}|^2+y^2}}}\Big|^2$; ∴

$$\overline{\frac{y}{n}-\frac{\overline{1-n}\times ry}{n\sqrt{\overline{d-x}|^2+y^2}}}\Big|^2=\frac{b^2}{a^2}\times a^2-\overline{\frac{\overline{d-x}}{n}-\frac{\overline{r-rn}\times\overline{d-x}}{n\sqrt{\overline{d-x}|^2+y^2}}}\Big|^2,$$

the Equation of the Curve *beap*; which, as it returns into itſelf by the Nature of the Section, is of the oval Kind.

Corollary.

Hence it appears, that, if $r=0$, the above Equation becomes $\frac{b^2}{a^2}\times$

a^2

namely, *if in any Priſmoid another ſtrait-ſided Veſſel (or Solid) be inſcribed, whoſe Ends are Ellipſes, the Tranſverſe and conjugate Diameters of each, reſpectively equal to the Lengths and Breadths of the ſaid Priſmoid; that then the Sections parallel to the Ends*

$\overline{a^2 - \frac{\overline{d-x}|^2}{n^2}} = \frac{y^2}{n^2}$, or, by ſubſtituting $\frac{d^2}{a^2}$ for n^2 (becauſe, in that Caſe, $na = d$, for $n : 1 :: d-r : a-r$), we ſhall then have $\frac{b^2}{a^2} \times \overline{a^2 - \overline{d-x}|^2} \times \frac{a^2}{d^2} = \frac{a^2}{d^2} \times y^2$, $\therefore y^2 = \frac{b^2}{a^2} \times \overline{2dx - x^2}$, anſwering to the Property of an Ellipſis, and ſimilar to the given One ABCD: But if $n = 1$, or mn coincides with EG, then $d = a$, and $\therefore$ the laſt Equation becomes $y^2 = \frac{b^2}{a^2} \times \overline{2ax - x^2}$, anſwering to the given Ellipſis ABCD.

SCHOLIUM.

The Nature of the Curve *beap*, is the very ſame as that which may be conceived to be deſcribed about an Ellipſis, ſimilar to the given one ABCD, in ſuch a Manner, that the Diſtance between the two Curves, meaſured in the Radius-Vector, may every-where be equal to a conſtant Quantity: For let OE (*Fig.* VI.) be $= R$ and $bE = R - r$; then $1 : n :: R - r : \overline{R-r} \times n = bm$; $\therefore nR - rn + r$ $(= nR + \overline{1-n} \times r) =$ Om the Radius-Vector; whence it is plain that the firſt Term (nR) expreſſes the Radius of an Ellipſis ſimilar to ABCD, and the ſecond Term ($\overline{1-n} \times r$) is a conſtant Quantity: But it may be proved in a general Manner, ſuppoſing two concentric Ellipſes deſcribed, having the Difference of the Conjugates equal to the Difference of their tranſverſe Diameters, that the Diſtance between the two elliptic Arcs, meaſured in any other Diameter, is not equal to the Difference of the Semi-conjugates, or Semi-tranſverſes.

By the ſame Method of Reaſoning, as in the preceding general Inveſtigation, it will be found that the Figure of the Section, parallel to the Ends of the Solid, can *never* be an Ellipſis, unleſs the ſaid parallel Ends were ſimilar Ellipſes, and ſimilarly poſited; *viz.* the Tranſverſe and conjugate Diameter of each End, reſpectively parallel to one another; which Circumſtance can only obtain when the Solid is the Fruſtum of an elliptic Cone: I ſhall only farther add, that the Curve (forming the Figure of the Section) will be of the ſame Order, whether the Ends of the Solid are diſſimilar Ellipſes, and ſimilarly poſited; or ſimilar Ellipſes, and diſſimilarly poſited; *viz.* the tranſverſe Diameter of one End parallel to the Conjugate of the other: But if the parallel Ends of the Solid are diſſimilar Ellipſes, and ſo poſited that neither the Tranſverſe nor conjugate Diameter of one End is parallel to *theſe* of the other End; then the Equation of the Curve, of the forementioned Section will be the moſt complex.

of ſuch a Solid, will alſo be Ellipſes:—But, that this cannot be the Caſe, the preceding *Note* will (I make no Doubt) ſufficiently convince every judicious Reader.—It may, however, be proper to obſerve, that the Content of a Tun of this Form may be obtained with the greateſt Expedition, by the general Rule laid down farther on (derived from the Method of equidiſtant Ordinates), and the Reſult will be ſufficiently exact, if the Section in the Middle be conſidered as an Ellipſis; For the ſaid Rule is *ſtrictly true* in every *ſtrait-ſided* Veſſel; provided the Meaſure of the two Ends, and that of a parallel Section in the Middle, can be *truly* determined:—This will be demonſtrated farther on.

PROP. VII.

To find the Content of a Sphere, in Ale and Wine Gallons, and Malt Buſhels.

RULE.

The Cube of the Diameter of the Sphere being

$$\text{multiplied by}\left\{\begin{array}{l}.0018567\\ .0022666\\ .000243\end{array}\right\}\begin{array}{l}\text{for Ale}\\ \text{Wine Gall.}\\ \text{Malt Buſh.}\end{array}\right\}\text{or divided}$$

$$\text{by}\left\{\begin{array}{r}538.58\\ 441.17\\ 4107.00\end{array}\right\}\begin{array}{l}\text{for Ale Gall.}\\ \text{Wine Gall.}\\ \text{Malt Buſhels}\end{array}\right\}\text{, will give the Con-}$$

tent ſought.

EXAMPLE.

Required the Content of a Sphere (in Ale Gallons, *&c.*) whoſe Diameter is 48 Inches.

OPERATION.

OPERATION.

$$\begin{array}{r} 48\\ 48\\ \hline 384\\ 192\\ \hline 2304\\ 48\\ \hline 18432\\ 9216\\ \hline 110592\\ \hline \end{array}$$

The Cube of the Diam. 110592

538.58)110592.0000(205.34 Ale Gallons.
441.17)110592.0000(250.22 Wine Gallons.
4107)110592.00 (26.92 Malt Bnſhels.

By the Sliding-Rule.

To $\left\{\begin{matrix}23.2\\21.0\\64.1\end{matrix}\right\}$ on D, ſet 48 on C; then againſt 48 on D, we ſhall have $\left\{\begin{matrix}295.34\\250.22\\26.92\end{matrix}\right\}$ the above Contents on C.

Becauſe the Content of every Sphere is $\frac{2}{3}$ds of its circumſcribing Cylinder, it is evident that the above Multipliers muſt be only $\frac{2}{3}$ds of thoſe for the Cylinder (ſee *Pa.* 83), and conſequently the Diviſors for the Cylinder, muſt likewiſe be $\frac{2}{3}$ds of thoſe for the Sphere.

There is another Way (beſides that given above) of ſolving this Queſtion by the *Sliding-Rule*, deduced from *Prop*. 6, *Pa*. 50, to which I refer; and only obſerve here, that the three given Numbers (in this Caſe) are

1, 48 and .0018567 } for Ale Gallons.
1, 48 and .002266 } Wine Gallons.
1, 48 and .000243 } Malt Buſhels.

Prop. VIII.

The Tranſverſe and Conjugate Axes of a Spheroid being given; to find its Content in Ale Gallons, &c. (ſee Def. 29, *Pa*. 59.)

Rule.

Multiply the Square of the Conjugate, by the tranſverſe Axis; and that Product being multiplied, or divided, as in the laſt Propoſition, gives the Content ſought.

Example.

Let the conjugate Axis of a Spheroid be 24.5 and the Tranſverſe 38 Inches; required its Content in Ale and Wine Gallons, and Malt Buſhels.

Operation.

Operation.

$$\begin{array}{lr} & 24.5\\ & 24.5\\ \hline & 1225\\ & 980\\ & 490\\ \hline \text{The Sq. of the Conjugate} & 600.25\\ \text{Transverſe} & 38\\ \hline & 480200\\ & 180075\\ \hline \end{array}$$

538.58)22809.5000(42.35 AleGal.
441.17)22809.5000(51.7 W. Gal.
4107)22809.50 (5.55 M. Buſh.

Becauſe every Spheroid is equal to $\frac{2}{3}$ds of its circumſcribing Cylinder; therefore the Method of Operation is the ſame as that for the Sphere.

By the Sliding-Rule.

To $\left\{\begin{array}{c}23.2\\21.0\\64.1\end{array}\right\}$ on D, ſet 38 on C; then oppoſite 24.5 on D, we ſhall have $\left\{\begin{array}{c}42.35\\51.7\\5.55\end{array}\right\}$ the above Content on C.

Prop. IX.

The Altitude of the Segment of a Sphere, and the Diameter of its Baſe being given; to find its Content in Ale Gallons, &c.

Rule.

RULE.

To the Square of the Altitude of the Segment; add three times the Square of half the Diameter of its Baſe; multiply this Sum by the Altitude; and the Product being multiplied, or divided, as in the preceding Propoſitions, gives the Content required.[*]

EXAMPLE.

Suppoſe AB the Altitude of the Segment ACD to be 12, and CD the Diameter of the Baſe 32 Inches; required its Content in Ale and Wine Gallons, and Malt Buſhels.

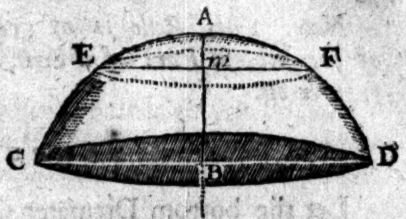
OPERATION.

---

[*] Let the Diameter CD (Fig. VII.) $= a$, $p = 3.1416$, CE $= x$, and DE $= a - x$; then EF^2 ($=$ ED $\times$ EC) $= ax - x^2$, whence the Fluxion of the Segment FCH is $pax\dot{x} - px^2\dot{x}$, the Fluent whereof is $\frac{pax^2}{2} - \frac{px^3}{3}$, or $px^2 \times \frac{3a-2x}{6}$, the Content of the Segment FCH: But, ſuppoſing EF to be denoted by d, we have, by ſimilar Triangles, $a \times x = x^2 + d^2$ (FC^2); $\therefore a = \frac{x^2+d^2}{x}$; which being ſubſtituted (above) for a, we get $p \times \frac{3x^3 + 3dx^2}{6} - \frac{2x^3}{6}$, or $\frac{px}{6} \times \overline{x^2 + 3d^2} =$ the Solidity of the Segment FCH; but $\frac{p}{6} = .5236$, $\therefore \frac{282}{.5236} = 538.58$, and $\frac{.5236}{282} = .0018567$. Q. E. I.

COROLLARY.

Hence, if x be taken $= a$ ($=$ the whole Diameter CD), then is $d = 0$; and $\therefore$ the above Expreſſion becomes $\frac{pa^3}{6} =$ the Content of the whole Sphere: But the Content of a Cylinder, whoſe Diameter is CD and the Height CE, is expreſſed by $\frac{pa^3}{4}$; whence it is evident that a Sphere is two-thirds of its circumſcribing Cylinder.



Operation.

The Square of the Altitude 144
3 times the Square of (16) the Semi-Baſe 768

Sum 912
Altitude 12

1824
912

10944

538.58)10944.0000(20.32 Ale Gallons.
441.17)10944.0000(24.80 Wine Gallons.
4107)10944.0000(2.66 Malt Buſhels.

Prop. X.

The top and bottom Diameters, and the Altitude of the Fruſtum of a Sphere being given; to find its Content in Ale Gallons, &c.

Rule.

To half the Sum of the Squares of the top and bottom Diameters, add $\frac{2}{3}$ds of the Square of the Altitude; this Sum being multiplied by the Altitude, and the Product divided by 359, 294, and 2738, will give the Content in Ale and Wine Gallons, and Malt Buſhels reſpectively.*

T *Note.*

---

* This Rule is very elegantly inveſtigated in *Simpſon*'s Fluxions, *1ſt Ed. Pa.* 211.

Note. This Rule is of great Uſe, in gauging of the globical Part of a Still, as will be exemplified farther on.

EXAMPLE.

Let the bottom Diameter CD (ſee the *preceding Figure)* be 34, the top Diameter EF 16.5, and the Altitude B*m* 7.2 Inches; to find the Content of the Fruſtum CEFD in Ale Gallons, *&c.*

OPERATION.

Bottom-Diam.	34	Top-Diam.	16.5	Alt.	7.2
	34		16.5		7.2
	136		825		144
	102		990		504
	1156		165		51.84
			272.25		2
			1156		103.68
		The Sum of the Squares of the top and bottom Diam.	1428.25		34.56

Half of which is 714.125
⅔ds of the Square of 7.2 is 34.56

Sum 748.685
Altitude 7.2

1497370
5240795

Product 5390.5320
359)5390.53(15.01 Ale Gallons.
294)5390.53(18.33 Wine Gallons.
2738)5390.53(1.96 Malt Buſhels.

PROP.

Prop. XI.

The tranſverſe and conjugate Axes of a Spheroid being given, and alſo the Height of a Segment thereof; to find its Content in Ale Gallons, &c. (*ſee* Def. 32, Pa. 60.)

Rule.

Divide the Product, contained under the conjugate Axis and the Altitude of the Segment, by the tranſverſe Axis, multiply the Square of that Quotient by the Difference between three times the tranſverſe Axis and twice the Altitude of the Segment; this Product being multiplied, or divided, as in the Sphere (*Pa.* 132), gives the Content ſought.*

Example.

Suppoſe the tranſverſe Axis AB of a Spheroid to be 80, the Conjugate CD 60, and the Altitude

---

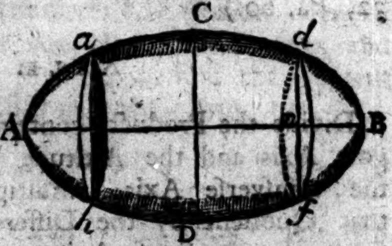
* It is proved, by the Writers on Fluxions, if the Diameter (or Axis) about which the Spheroid is ſuppoſed to be generated, be put $= a$, the other Diameter $= b$, and $p = 3.1416$, that the Meaſure of a Segment, whoſe Altitude is $= x$, will be expreſſed by $\frac{pb^2}{a^2} \times \overline{\frac{ax^2}{2} - \frac{x^3}{3}}$, or $\frac{p}{6} \times \frac{b^2x^2}{a^2} \times \overline{3a-2x}$: But $\frac{p}{6} = .5236$, and $\frac{.5236}{282} = .0018567$; therefore $.0018567 \times \overline{\frac{bx}{a}}\Big|^2 \times \overline{3a-2x}$ (or $\frac{\overline{\frac{bx}{a}}\Big|^2 \times \overline{3a-2x}}{538.58}$) will expreſs the Meaſure of a Segment in Ale Gallons. Q. E. I.

Corollary.

If x be taken $= a$, we ſhall have $.001857 \times ab^2$ for the Meaſure of the whole Spheroid, which is two-thirds of $.0027851 \times ab^2$, the Meaſure of a Cylinder (in Ale-Gallons) whoſe Diameter is b, and Altitude a.

B*n*, of the Segment *d*B*f*, to be 16 Inches; required its Content in Ale and Wine Gallons, and Malt Buſhels.

OPERATION.



Alt. of the Seg. 16
Conjugate 60

80)960(12
80

160
160

. .

Three times AB is 240
Twice B*n* is 32

Difference 208
Multiply by the Sq. of 12 (found above) 144

832
832
208

Product is 29952

538.58)29952.0000(55.61 Ale Gallons.
441.17)29952.0000(67.91 Wine Gallons.
4107)29952.00 (7.29 Malt Buſhels.

PROP. XII.

The Altitude, and the Diameter of the Baſe of a parabolic Conoid being given; to find its Content in Ale Gallons, &c. (*ſee* Def. 30, Pa. 59.)

RULE.

Rule.

The Square of the Diameter of the Baſe, being multiplied by half the Altitude; and the Product divided by 359, 294, and 2738, gives the Content in Ale and Wine Gallons, and Malt Buſhels reſpectively.

Example.

Suppoſe the Diameter of the Baſe to be 68.5, and the Altitude 42 Inches; required the Content in Ale Gallons, *&c.*

Operation.

68.5
68.5

3425
5480
4110

The Square of the Diam. 4692.25
$\frac{1}{2}$ the Altitude . . . 21

469225
938450

359)98537.25(274.47 Ale Gal.
294)98537.25(335.16 W. Gal.
2738)98537.25(35.98 M. Buſhels.

Prop. XIII.

The top and bottom Diameters, and the Altitude of the Fruſtum of a parabolic Conoid being given; to find its Content in Ale Gallons, &c.

Rule.

RULE.

The Sum of the Squares of the top and bottom Diameters being multiplied by half the given Altitude, and the Product divided by 359, 294, and 2738, gives the Content in Ale and Wine Gallons, and Malt Buſhels reſpectively.

EXAMPLE.

Let the greater Diameter of the Fruſtum of a parabolic Conoid be 45, the leſſer Diameter 27, and the Altitude 40 Inches; what is the Content thereof in Ale Gallons, *&c.*

OPERATION.

45	27
45	27
225	189
180	54
2025	729
729	

The Sum of the Squares of the Diameters } 2754

½ the Altitude 20

55080

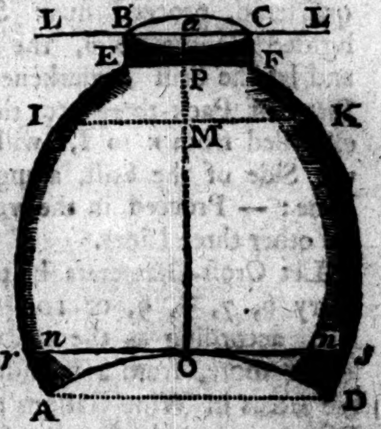
359)55080.00(153.42 Ale Gallons.

294)55080.00(187.34 Wine Gallons.

2738)55080.00(20.11 Malt Buſhels.

To

To take the Dimenſions of a STILL, *and to find its Content in Wine Gallons; and alſo a Method for tabulating the ſame, to every Inch of its whole Depth.*



Suppoſe ABCDO to repreſent a Still, when fixed, and A the Place of the Cock; upon O, the Center of the Crown, (which is eaſily diſtinguiſhed, by a ſmall Dent made by the Copper-Smith), apply the *Sliding-Piece** ſo that the perpendicular Diſtances A*n* and D*n* may be equal to each other, and let Marks be made at *r* and *s*, as already deſcribed in the Gauging of a Copper, *Pa.* 125; then lay a ſtrait Rule (or Rod LL) diametrically over the Top of the Collar of the Still, and with the End of the Dimenſion-Cane on the Center O, find the neareſt Diſtance to the ſaid Rule LL, and that Diſtance (*viz. a*O) will be the whole Length, from which, the Depth of the Collar being ſubtracted, we ſhall obtain PO the internal Length of the Still: Now, the Dimenſion-Cane being kept exactly in the Poſition *a*O, with the Help of your Aſſiſtant, let the Plumb-Line be extended as a Diameter IK, exactly at the Seam which is formed by joining the globical Part to the Body of the Still; and meaſure the Diſtance MO, which being taken from the whole internal Length PO, leaves PM the Altitude of the globical Part IEFK.

Quarter the Still at the Bottom, and alſo at the Altitude OM, by the Method laid down (*Pa.* 117) for a circular Veſſel; then in Order to draw four chalk Lines up the Sides of the Still, ſo that it may be every-where truly qnartered, proceed thus: Stick a ſmall Piece of lighted Candle at *m*, the Center of the Crown, and let the Still be darkened at the Top; then a Piece of Pack-thread (or the Plumb-Line) being extended from *r* to I, will form a Shadow on the Side of the Still, along which draw a chalk Line: — Proceed in the very ſame Manner for the other three Lines.

Let Croſs-Diameters be taken in the Middle of every 6, 7, 8, 9, or 10, *&c.* Inches, more or leſs, according as the Body of the Still differs, leſs or more, from a cylindrical Form; then find the Areas in Wine Gallons correſponding to theſe Diameters, multiply the Sum of the Areas by their common Diſtance aſunder (ſee *Pa.* 116), and the Product will give the Content of the Body of the Still *rIKs*; to which add the Content of the globical Part IEFK (found by *Prop.* 10. *Pa.* 137), and alſo the Quantity which *exactly* covers the Crown, and we ſhall then obtain the whole Content of the Still AEFDO.

Note. It will always be proper to take an even Number of Areas in every Veſſel, whoſe greateſt Diameter is at the Middle of its Length, ſuch as *Caſks*, *Stills*, *&c.* otherwiſe one of the Dimenſions will fall in the Middle, by which Means ſuch Veſſels would be over-gauged.

Note. I cannot but acknowledge my Obligations to Mr. *Thomas Stephens*,* General Surveyor of the LONDON *Diſtillery*, for the above Method of drawing the four chalk Lines up the Sides of the *Still*; and alſo for communicating to me, ſome uſeful and ingenious Hints, relative to taking the *Dimenſions* of this, and other Veſſels.

Let the Altitude PM of the globular Part (ſee the preceding Figure) be 9 Inches, and let the Croſs-Diameters at the Top and Bottom of the ſaid globular Part, and alſo *thoſe* taken in the Body of the Still be as follow.

	Depth.	*Croſs-Diameters.*		*W. Areas.*	*Gallons.*
Alt. PM	9.0	27.0 56.3	27.0 56.2	Content of the globular Part, found by *Prop.* 10. *Pa.* 137.	61.21
	9	59.1	60.3	12.11	108.99
	9	63.8	64.1	13.88	124.92
	9	64.2	64.5	14.05	126.45
	10.6	62.1	62.5	13.99	148.29
				To cover the Crown	36.00
DepthGM	46.6			The whole Content	605.86 W. Gallons.

Groſs Depth 49.8
Collar 3.2

Whole Depth 46.6
Altitude of the globical Part 9.0

Depth of the Body 37.6

Crown 36 Gallons.

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* The *Sliding-Piece (Pa.* 143, contrived by this Gentleman) is very uſeful in taking the Croſs-Diameters of a *Copper*, or *Still:* It conſiſts of two Rods ſliding one by the other, in the ſame Manner as a Pair of Calipers, and when drawn out its full Length (ſee *Fig.* VIII. in the Plate) is 62 Inches; on one Side are graduated Inches and Tenths, and on another are the correſponding Wine Areas; at about one and two Inches from each End (more or leſs, according to the Size of ſuch Inſtrument) are two equal ſquare Holes, to which are fitted two ſmall Pieces to ſlide therein, marked with Inches and Tenths from the Bottom (ſee the *Figure*); theſe ſerve to take a true Diameter directly upon the Center of the Crown, as *rs* (ſee the *Fig. Pa.* 143.)

The Method of tabulating the Body of a Still (or indeed any Veſſel which is ſuppoſed to have one common Area, or a certain Number of different ones) is the very ſame as that given for a Diſtiller's *Waſh-Back*, *Pa*. 119, to which it may be proper to refer. — But to determine the *true* Quantity upon every Inch of the globular Part, we muſt previouſly find the Square of the Semi-diameter of that Sphere to which the ſaid globular Part correſponds; in Order thereto, obſerve the following general

RULE.

Divide the Difference of the Squares of half the top and bottom Diameters by twice the Altitude of the Fruſtum, from the Quotient ſubtract half the ſaid Altitude, and the Remainder will be the Diſtance between the Middle of the greater Diameter and the Center of the Sphere;* the Square of

---

* Let PF ($= \frac{1}{2}$ the leſſer Diameter, ſee the following Fig.) $= a$, KM ($= \frac{1}{2}$ the greater Diameter) $= b$, the Altitude PM $= c$, and the required Diſtance OM $= x$; whence (by 47. *Eu.* 1.) $c^2 + 2cx + x^2 + a^2 = x^2 + b^2$, $\therefore x = \frac{b^2 - a^2 - c^2}{2c} = \frac{b^2 - a^2}{2c} - \frac{c}{2}$. Q. E. I.

LEMMA.

If the Terms of any arithmetical Progreſſion (either aſcending or deſcending) be ſquared, and diſpoſed of in a Series; then will the Differences of every two adjacent Terms of that Series, form another arithmetical Progreſſion, whereof the common Difference will be expreſſed by twice the Square of the common Difference of the firſt Progreſſion.

For any arithmetical Progreſſion, whoſe firſt Term is m, and the common Difference n, will be expreſſed by m, $+ m \pm n$, $+ m \pm 2n$, $+ m \pm 3n$, $+ m \pm 4n$, $+$ &c. whereof the Square of each Term is, m^2, $+ m^2 \pm 2mn + n^2$, $+ m^2 \pm 4mn + 4n^2$, $+ m^2 \pm 6mn + 9n^2$, $+ m^2 \pm 8mn + 16n^2$, $+$ &c. therefore

The Differences of the two adjacent Terms will form the following Series, *viz.* $\pm 2mn + n^2$, $\pm 2mn + 3n^2$, $\pm 2mn + 5n^2$, $\pm 2mn + 7n^2$, $+$ &c. the common Difference of which, is evidently $2n^2$. Q. E. I.

COROLLARY.

of which being added to the Square of half the greater Diameter, gives the Square of the Semidiameter of the Globe ſought. *(See the following Figure.)*

In the preceding Example EP (or PF) is 13.5

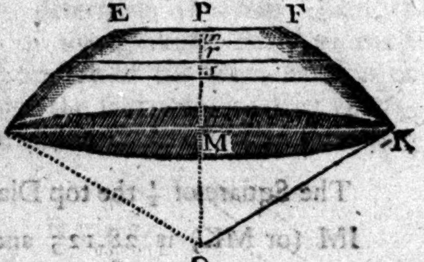
```
                                                  13.5
                                                  13.5
                                                -------
                                                   675
                                                  405
                                                 135
                                                -------
The Square of ½ the top Diameter                182.25

IM (or MK) is 28.125 and twice PM is 18.
              28.125
           -----------
               140625
               56250
              28125
            225000
            56250
           -----------
The Sq. of ½      }
the bott. Diam.   } 791.015625
                    182.25
           -----------------
        18)608.765625(33.82
                      4.5  = ½ PM.
                    -------
                    29.32 = MO.
          Add PM     9.00
                    -------
          Gives PO  38.32
```

Corollary.

If, inſtead of the Differences of the adjacent Terms of the 1ſt Progreſſion, we diſpoſe of the Differences of the 1ſt and 3d, the 2d and 4th, the 3d and 5th, the 4th and 6th, &c. Terms into a Series, we ſhall then have $\pm 4mn+4n^2$, $\pm 4mn+8n^2$, $\pm 4mn+12n^2$, $\pm 4mn+16n^2$, &c. the common Difference whereof (inſtead of $2n^2$) is manifeſtly $4n^2$; that is, four times the Square of the common Difference of the firſt Progreſſion.

Then the Square of 29.32 (MO) is 859.6624
Add the Square of 28,125 (KM) 791.0156

Gives the Sq. of the Semi-diam. OK (OI) 1650.6780



Having obtained the Square of the Semi-diameter of the Sphere, whereof the Segment IEFK is a Part; then, in Order to inch it down, obſerve the following Method.

1. To half the Square of the leſſer Diameter EF, add twice the Difference between the Square of the Semi-diameter OK, and the Square of O*m* (*viz.* PO leſſened by one Inch), to this Sum add .6666 *&c.* (*i. e.* $\frac{2}{3}$); multiply this laſt Sum by .0034, and the Product will be the true Content of the firſt Inch, in Wine Gallons.

2. From the Square of the Semi-diameter OK ſubtract the Square of O*r* (*viz.* PO leſs two Inches), and from twice the Remainder take half the Square of the top Diameter EF, multiply the Remainder by .0034; then add this *Product* to the Quantity upon the 1ſt Inch (found as above), the Sum will give the true Meaſure of the 2d Inch.

3. Let the laſt mentioned *Product* be *reſerved*, from which take .0272,† *reſerving* the *Difference*,

---

† Let I*ab*K (*Fig.* IX.) repreſent the Fruſtum of a Sphere, O its Center; draw OM perpendicular to *ab*; take M*m*=1, M*r*=2, MS=3, and Me

which being added to the Quantity contained in the 2d Inch (found as above), gives the Quantity for the 3d Inch: Again, from the *reſerved Difference* take .0272, add the Remainder to the 3d Inch, the Sum will give the Quantity for the 4th Inch; proceed in the ſame Manner to find the Quantity for the 5th, 6th, 7th, *&c.* to the laſt Inch of the globular Part. See the following

OPERATION.

$Mv = 4$ Inches: Then (by *Prop.* 10) the Meaſure, in Wine Gallons, of the 1ſt, 2d, 3d, and 4th, *&c.* Inch from the Top is expreſſed by

$$\overline{\frac{ab^2}{2}+\frac{cd^2}{2}+.6666}\times .0034.$$

$$\overline{\frac{cd^2}{2}+\frac{ef^2}{2}+.6666}\times .0034.$$

$$\overline{\frac{ef^2}{2}+\frac{gb^2}{2}+.6666}\times .0034.$$

$$\overline{\frac{gb^2}{2}+\frac{ik^2}{2}+.6666}\times .0034.$$

&c.

Hence it appears, that the *accurate* Increaſes of the 2d Area *(ecdf)* from the 1ſt, the 3d from the 2d, and the 4th from the 3d Area, *&c.* are reſpectively equal to the Increaſes of half the Square of the 3d Diameter *(ef)* from the 1ſt *(ab)*, the 4th *(gb)* from the 2d *(cd)*, and the 5th *(ik)* from half the Square of the 3d *(ef)*; or, which is the ſame Thing, the Increaſes of the 2d Area from the 1ſt, the 3d from the 2d, the 4th from the 3d, *&c.* are equal to the Decreaſes of $2Or^2$ from $2OM^2$, $2Os^2$ from $2Om^2$, *&c.* Now, by the preceding *Corollary*, $2OM^2 - 2\times\overline{OM-2}|^2$ *leſs* $2\times\overline{OM-1}|^2 - 2\times\overline{OM-3}|^2$ is $= 8$ (becauſe n, in this Caſe, $= 2$, and $\therefore 2\times 2n^2 = 8$); conſequently the Difference between $2\times\overline{OM^2 - Or^2}$ and $2\times\overline{OM^2 - Os^2}$, or (which is ſtill the ſame) the Difference between $2\times\overline{rf^2 - Mb^2}$ $(\frac{ef^2}{2}-\frac{ab^2}{2})$ and $2\times\overline{sh^2 - md^2}$ $(\frac{gb^2}{2}-\frac{cd^2}{2}) = 8$: Which being multiplied by .0034 (in Order to reduce it to Wine Meaſure), gives .0272, the common Addend, or Subtrahend, according as we begin to inch the Fruſtum, at its greater or leſſer End. Q. E. I.

OPERATION.

Half the Square of 27 (the leſſer Diameter EF) is	364.5
The Square of the Semi-diameter OK (found above) is - -	1650.6780
The Square of 37.32 (O*m*) is	1392.7824
Difference	257.8956
Multiply by	2
	515.7912
Add	364.5
	880.2912
Add	.6666
	880.9578
Multiplied by	.0034
	35238312
	26428734
Gives the *true* Quantity, for the 1ſt Inch from the Top	2.99525652

The Sq. of the Semi-diam. (found above) is } 1650.6780
Sub. the Squ. of 36.32 (Or) 1319.1424

Remainder 331.5356

The Double of which is 663.0712
Subtr. ½ the Square of 27 (EF) 364.5

Remainder 298.5712
.0034

119428
89571

Reserved Product 1.015138 add to the [1st Inch.
Subtract .0272

Inch.	*W. Gallons.*	*Reserved Dif.*
1	2.9952 Add 1.0151	.9879 add to the Subtract .0272 [2d Inch.
2	4.0103 Add .9879	.9607 add to the Sub. .0272 [3d Inch.
3	4.9982 Add .9607	.9335 add, &c. Sub. .0272
4	5.9589 Add .9335	.9063 Sub. .0272
5	6.8924 Add .9063	.8791 Sub. .0272
6	7.7987 Add .8791	.8519 Sub. .0272
7	8.6778 Add .8519	.8247
8	9.5297 Add .8247	
9	10.3544	

It muſt be allowed that, in the preceding Method of gauging a Still, a very ſmall Error may ariſe, on Account of a little Inclination which is uſually given to it, when fixed, as was obſerved in *Pa.* 124: Nor indeed is there any Method for gauging an inclined Still, that I know of, but what is liable to ſome Objection.—For, even ſuppoſing we take the Croſs-Diameters parallel to the Horizon, and conſider the Surface of the Liquor, in any Part of the Body of the Still, to form an Ellipſis (inſtead of a Circle), we ſhall then find that the Content of the globular Part of the Still cannot be truly determined by the general Rule given for that Purpoſe: Beſides, the Line PO (ſee the *Fig. Pa.* 143) will not, in that Caſe, be the *true Depth* of the Still; for *that* will be repreſented by the perpendicular Diſtance of two horizontal Planes, one paſſing through the higheſt Point in the Crown, and the other through the loweſt Point at the Top of the Still; which Dimenſion, though differing but *very little* from the Diſtance PO (*vid. Fig. Pa.* 143), ought to be *truly* known; but *that*, indeed, would be very difficult (if not impracticable) to effect.

It may be proper to obſerve, that, in tabulating the whole Content of a Still, much Labour will be avoided, if the Altitude of the globular Part be taken a whole Number, and the Decimal Parts (if any happen in the whole Depth) be conſidered in the bottom Area: See *Pa.* 145.

Some Authors conſider the riſing Crown of a Copper, or Still, in the Form of the *Segment* of a *Sphere*, and alſo the Part ArOsDA (ſee the *Fig. Pa.* 143) as the Fruſtum of a *Parabolic Conoid* or *Cone*; and therefore the Quantity of Liquor to cover the Crown will then be determined by the foregoing *Prop.* *viz.* by ſubtracting the Meaſure of the Part AODA from that of A*r*O*s*DA: — But, on Account of the Difficulty of obtaining the *true* Diameter and Altitude of the Crown (even admitting the two Figures to be as above repreſented), I apprehend that *that* Quantity may be found, with much more Certainty and Expedition, by covering (as exact as poſſible) the higheſt Point of the Crown with Water, and then carefully drawing off the ſame, into a Veſſel whoſe Meaſure is *truly* known.

Note. It very frequently happens the Depth (or Altitude) of a Veſſel is ſuch, that the Croſs-Diameters, *&c.* cannot all be taken at equal Diſtances from each other; or, which comes to the ſame, the ſaid Depth cannot be divided, without a Remainder, by the Number of Areas neceſſary (and ſufficient) to be taken: In that Circumſtance, I apprehend, it will be the beſt Way to conſider ſuch Remainder in the uppermoſt Area, as that Part of the Veſſel will be the leaſt ſubject to cauſe an Error, in any Charge which may ariſe from it; not only becauſe the Surface of the Liquor ſeldom reaches that Area, but alſo becauſe ſtrait-ſided Veſſels (as *Guile-Tuns*, *Waſh-Backs*, *&c.)* generally ſtand upon their greater Ends: See *Pa.* 119.

SECTION IX.

OF CASK-GAUGING.

IT has been a general Cuſtom, with Authors on this Subject, to include among the Varieties of Caſks, thoſe of the following Denominations; namely, the Fruſtums of two Parabolic Conoids, and Cones, each of theſe abutting (as it is uſually termed) upon one common Baſe.

But it is well known, from common Experience, that every cloſe Caſk, whether *Pipe*, *Butt*, *Hogſhead*, *&c.* and of what Variety ſoever, is always found to have a Continuity of Curvature at the Bulge, and not to form there an Angle (or ſharp Ridge), which will be actually the Caſe, if we conceive a Caſk to be formed either of two Fruſtums of *Parabolic* (or *Hyperbolic*) *Conoids*, or the Fruſtums of two *Cones:* Therefore, as no ſuch Caſks as theſe are ever made, it cannot, I preſume, be deemed a Crime to expunge thoſe two Varieties; as they have hitherto only embarraſſed the Subject, puzzled the Learner, and even rendered every Perſon, concerned in Caſk-Gauging, more liable to fall into Error.

There is another conſiderable Imperfection in this Branch of Gauging, of which it may be proper to take Notice.

It has been aſſerted by many Authors, who have treated on this Subject, that there is no Rule, or Method, can be given, whereby a Perſon can, with any Degree of Certainty, determine the Variety of the Caſk; that is, whether a Caſk is in the Form of the Middle Fruſtum of a *Spheroid*, *Parabolic Spindle*, or *Hyperbolic Spindle*.

It is true, indeed, no Rules can be given for determining the *exact* Form, or Variety, of the Caſk; yet I preſume thoſe which I am going to offer, if duly attended to, will be found of ſingular Uſe, as they will readily diſcover to us, what Variety any Caſk, *very nearly*, approaches to; that is, whether the Caſk may be taken as the Middle Fruſtum of a *Spheroid*, or of a *Parabolic* or a *Hyperbolic Spindle*.

Some Authors direct us to judge from Experience of the Variety of the Caſk: Others divide the Difference between the ſpheroidical Caſk, and that compoſed of the Fruſtums of two Cones, into three, or four, equal Parts; and then attempt to lay down Rules for determining theſe different Varieties.

But (even admitting it poſſible that a cloſe Caſk could be formed of the Fruſtums of two Cones) theſe Rules appear to be arbitrary, and to have no Foundation in Science; and likewiſe ſeem to be derived from a Suppoſition that *all* ſpheroidical Caſks are the Middle Fruſtums of ſuch Spheroids, whoſe Tranſverſe and conjugate Axes are in ſome conſtant Proportion; or, which amounts to the ſame, that every ſpheroidical Caſk has the ſame Degree of Curvature; but a very ſmall Knowledge in Conic Sections will be ſufficient to convince any One, that there are a vaſt Number of different Forms of Ellipſes, and conſequently Spheroids: — I thought it would not be improper to mention this laſt Circumſtance, in Order to rectify an Error which ſome are apt to fall into, by imagining thoſe Caſks are not of a ſpheroidical Form, which *appear* to have but little Curvature, or whoſe Bung and Head Diameters are nearly equal to each other.

Although it may be ſaid, that the following Method is too tedious for ordinary Practice, or for the Officer to aſcertain by it, the Variety of all the different Caſks which daily fall under his Inſpection; yet I dare venture to affirm, that whoever will take the Pains to make themſelves acquainted with the following Directions, will not only be able to diſtinguiſh, *very nearly*, the true Variety of the Caſk; but will, moreover, have a better Idea of it, even by Inſpection, than by any Method hitherto delivered for that Purpoſe.

The different Forms of Caſks, with Regard to Curvature, may be juſtly comprehended under theſe four Denominations:

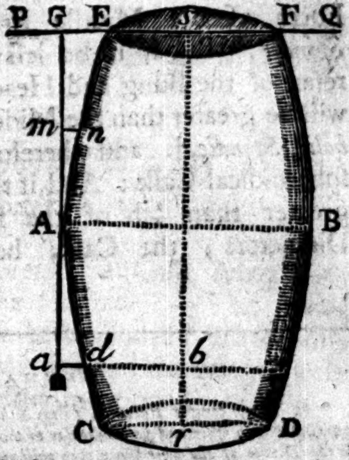
Viz. The	*Elliptic Spindle.*	———
Middle	*Spheroid*	1ſt Variety.
Fruſtum	*Parabolic Spindle*	2d Variety.
of the	*Hyperbolic Spindle*	3d Variety.

But as it very ſeldom (if ever) happens, that a cloſe Caſk is found to contain more than the Middle Fruſtum of a Spheroid; it will therefore be unneceſſary to give any Examples of the elliptic Spindle; — And I have purpoſely omitted the circular Spindle, on Account of the near Affinity it bears to the ſpheroidical Caſk; beſides, the Rule for determining its Content is far too intricate for practical Uſe.

To take the Dimenſions of a ſtanding Caſk; and an expeditious, general, Method of determining, very nearly, the true Variety thereof.

Let AEFBDCA repreſent a Caſk ſtanding upon one Head, with its Axis *(sr)* perpendicular to the Horizon: — Firſt take with your Rule the Diſtance from the Inſide of the Chime (cloſe to the Head) to the Middle of the ſlope Edge (or Thickneſs)

Thickneſs) of the oppoſite Staff; that Diſtance will meaſure the Head-Diameter within the Caſk, *very nearly*.



Then lay any ſtrait Rule (or Rod PQ) on the Top of the Caſk, paſſing over the Center (*s*) of the Head, and let a Plumb-line, with a Nooſe at one End, be ſlid backward and forward on the Rod (PQ), 'till it juſt touches the Bulge of the Caſk at A; meaſure, carefully, the Diſtances from the Nooſe (G) to the out-ſide of each Chime at E and F, and from their Sum (*viz.* the Sum of GF and GE) take twice the Thickneſs of a Staff at the Bulge of the Caſk (as your Judgment directs, according to the Size of the Caſk) and the Difference (AB) will be the Bung-Diameter required.

It is unneceſſary to give any Directions for taking the Length of a Caſk in this Poſition, ſuppoſing there to be a Hole in the top Head.

But, to determine the Variety of the Caſk, proceed thus: — Let $\frac{1}{4}$th of the internal Length of the Caſk be ſet off, from the Bulge (A) towards either Head, on the Plumb-Line (GAW), or rather, upon any ſtrait Rod, placed exactly in that Poſition; that is, let A*m* be equal to $\frac{1}{4}$th of the internal Length of the Caſk: Then if the perpendicular Diſtance (*mn*) from the Rod to the Caſk, be equal to $\frac{1}{8}$th of the Difference between the Bung and Head Diameters, the Caſk is then extremely near (if not exactly) the

Form of the Middle Fruſtum of a *Parabolic Spindle:** But if it be leſs than $\frac{1}{8}$th of the Difference of the Bung and Head Diameters; the Caſk will be greater than the Middle Fruſtum of a *Parabolic Spindle*,† and therefore may be taken as a ſpheroidical Caſk: And if the ſaid Diſtance (*mn*) is greater than $\frac{1}{8}$th of the Difference of the ſaid Diameters; the Caſk, being then leſs than a

Parabolic

LEMMA I.

* *In every conical Parabola (Fig. X.) if any two Ordinates be drawn parallel to each other and perpendicular to the Axis* CQ, *and ſo that one* AE *may be the Double of the other* Hb; *then will one Abſciſſa* CE, *be* exactly *equal to four times the other Abſciſſa* C*b*.

For, by the Property of the Curve, C*b* : CE :: Hb^2 : AE^2; but, *by Hypotheſis*, AE $=$ 2H*b*; $\therefore$ AE$^2 = 4$Hb^2; conſequently C*b* : CE (:: Hb^2 : 4Hb^2) :: 1 : 4. Q. E. I.

LEMMA 2.

† *In every Ellipſis (Fig. XI.) if any two Ordinates be drawn parallel to the tranſverſe Axis, and in ſuch a Manner, that one* EF, *may be juſt the Double of the other* Hb; *then, I ſay that the greater Abſciſſa* CF, *will be, always, more than four Times the leſſer Abſciſſa* C*b*.

Let the tranſverſe and conjugate Diameters, of any Ellipſis, be denoted by *m* and *n* reſpectively; alſo let C*b* $= x$ and CF $= y$; then, by the Property of the Curve, we have $\overline{nx-x^2} \times \frac{m^2}{n^2} = \mathrm{H}b^2$, and alſo $\overline{ny-y^2} \times \frac{m^2}{n^2} = \mathrm{EF}^2$; but, *by Hypotheſis*, EF $=$ 2H*b*; $\therefore$ EF$^2 = 4$Hb^2, and therefore $4nx-4x^2=ny-y^2$: Hence it is very plain, that if *y* be taken equal to (or leſs than) 4*x*, the above Equation is impoſſible; for it becomes (by ſubſtituting 4*x* for *y*) $4nx-4x^2=4nx-16x^2$, or $x=2x$, which is *abſurd*; but if, inſtead of *y*, *dx* be wrote in the above general Equation, ſuppoſing *d* to repreſent any Number (whole or broken) greater than 4; then the ſaid Equation becomes a poſſible One, from whence the Value of *x* (and that of *y*) may be determined. Q. E. I.

Hence it appears, that the above Property obtains in a Circle; that is, if in any Circle, two parallel Chords be ſo drawn, that one is the Double of the other; then the verſed Sine of the greater Segment, will always be more than four times the verſed Sine of the Leſſer: The Truth of which may be, eaſily, made out, from a Conſideration independent of Algebra.

Parabolic Spindle,[‡] may be conſidered of the 3d Variety, or the Middle Fruſtum of an hyperbolic Spindle.

To take the Dimenſions, &c. of a lying Caſk.

Let ACDBFEA repreſent a Caſk lying with its Axis parallel to the Horizon: — The Head and Bung Diameters are here obtained in the ſame Manner, as the Head Diameter and the Length were in the ſtanding Caſk before-mentioned.

The moſt expeditious Way of taking the Length is, with a Pair of Calipers; but as it cannot be expected that every One, concerned in the Art of Gauging, is furniſhed with this Inſtrument; I ſhall therefore lay down the following Method.

Apply any ſtrait Rod (PQ) to the Bulge of the Caſk, in ſuch a Poſition, that a Plumb-Line, being

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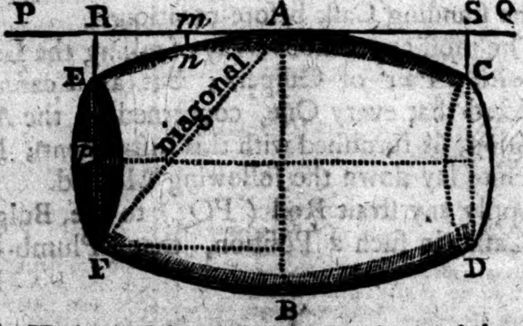
LEMMA 3.

[‡] *If two Ordinates (Fig. XII.) be drawn in any Hyperbola, perpendicular to the Axis* CQ, *ſo that the one* EG, *may be juſt the Double of the other* eb; *then will the leſſer Abſciſſa* Cb, *be always more than* $\frac{1}{4}$*th of the greater Abſciſſa* CG, *and leſs than one half thereof.*

Let the tranſverſe and conjugate Diameters of any Hyperbola, be denoted by m and n reſpectively; alſo let $Cb = x$, and $CG = y$; then, by the Property of the Curve, we have $\overline{mx + x^2} \times \frac{n^2}{m^2} = eb^2$, and likewiſe $\overline{my + y^2} \times \frac{n^2}{m^2} = EG^2$; but, *by Hypotheſis*, $EG = 2eb$; $\therefore EG^2 = 4eb^2$, conſequently $my + y^2 = 4mx + 4x^2$: Hence it is evident, if, inſtead of y, there be wrote dx, (for $1 : d :: x : y$), the above general Equation becomes $4mx + 4x^2 = dmx + d^2x^2$, or $4m + 4x = dm + d^2x$, whence $x = \frac{4m - dm}{d^2 - 4}$; hence the Value of x may be found, provided d repreſents any Number, (whole or broken) leſs than 4 and greater than 2; otherwiſe, it is very plain, the Equation would be *abſurd*: Hence alſo the Truth of the *Lemma* is manifeſt. Q. E. I.

being ſuſpended on the Rod, may paſs over (*p*) the Center of the Head, and obſerve to keep the Rod equally diſtant from each Chime of the Caſk: This done, and the Rod being kept exactly in this Poſition, lay a ſtrait Rule acroſs each End of the Caſk, to meet the top Rod in R and S; then let the Diſtance RS be carefully meaſured, from which ſubtract the Depths of each Chime, together with the Thickneſs of the Heads (as your Judgment directs), the Remainder will be the internal Length of the Caſk.



The Variety of the Caſk may be obtained, by meaſuring the perpendicular Diſtance *mn*, and proceeding in the very ſame Manner, as above directed.

Though it is demonſtrable the Property of every ſpheroidical Caſk is ſuch, that the Diſtance *mn* (ſee the Figure) may be any Quantity leſs than $\frac{1}{8}$th of the Difference between the Bung and Head Diameters; nevertheleſs, as various Curves may be deſcribed through the ſame three Points, this Property may hold good (with Regard to thoſe Points), and yet the Caſk may, perhaps, be a *ſmall* Matter either greater or leſs, than the Middle Fruſtum of a Spheroid; in which Form it may, however, always be taken, under the above Circumſtance, without ſenſible Error: — The ſame is to be obſerved, with Reſpect to the other two Varieties.

Perhaps, ſome Readers may look upon this Method of determining the Varieties of Caſks, as a Matter of Speculation only, and not to be regarded in Practice; but, I apprehend, its Utility will ſo remarkably appear, in the following Examples, particularly in finding the Contents of large Caſks, as ſufficiently to obviate all Objections on that Head.

EXAMPLE I.

Let it be propoſed to find the Content of a Caſk, in Ale and Wine Gallons; whoſe Bung Diameter is 32, Head Diameter 24, and the Length 42 Inches.

Suppoſe, by proceeding according to the foregoing Directions, the Diſtance *mn* (ſee the laſt *Fig.*) was found to be ſomething leſs than one Inch (*i. e.* leſs than $\frac{1}{8}$th of the Difference of the Bung and Head-Diameters); conſequently, this Caſk, having the ſame Property as every *Spheroidical Caſk*, muſt be gauged as ſuch by the following general

RULE.

To twice the Square of the Bung, add once the Square of the Head-Diameter; this Sum being multiplied by the Length, and the Product divided by 1077.15 (*viz.* three times 359.05) for Ale, or by 882.36 (three times 294.12) for Wine Gallons, will give the Content required.

OPERATION.

Bung-Diameter 32 Head-Diameter 24
32 24

64 96
96 48

1024 Sq. of the H. Diam. 576
2

Twice the Square of the Bung-Diameter 2048
576

Sum 2624
Which multiplied by the Length 42

5248
10496

1077.15)110208(102.31 = the Content in Ale Gallons.

And 110208 being divided by 882.36, the Quotient will be 124.9, the Content in Wine Gallons.

Note. In Practice we may reject the above Decimals in the Diviſors, without any material Error in the Reſult.

EXAMPLE 2.

Wherein it is propoſed to find the Content of a Caſk in Wine Gallons, whoſe Bung-Diameter is 65, Head-Diameter 39, and the Length 110 Inches.

Suppoſe, by the Directions given in *Pa.* 157, the Diſtance *mn* (ſee either of the preceding *Figures*) was found to be 3 Inches, which is leſs than $\frac{1}{8}$th of (26) the Difference of the Diameters; therefore this Caſk, having the Property of every *Spheroidical Caſk*, muſt alſo be gauged by the laſt *general Rule.*

OPERATION.

Bung-Diameter 65 Head-Diameter 39
65 39
325 351
390 117
The Sq. of the B. Diam. 4225 Sq. of the H. Diam. 1521
The Double is 8450
Add 1521
Sum 9971
Multiplied by the Length 110
99710
9971
882)1096810(1243.54 = the Content in Wine Gallons.

EXAMPLE 3.

Let it be required to find the Content of a Caſk in Ale and Wine Gallons, having the ſame Length, Bung and Head-Diameters, as that in the 1ſt Example; only let the Diſtance *mn* be ſuppoſed equal to one Inch.

Here, the Diſtance *mn* (ſee the preceding *Fig.*) being equal to $\frac{1}{8}$th of the Difference of the Head and Bung-Diameters; this Caſk, therefore, having the very ſame Property as the Middle Fruſtum of every *Parabolic Spindle*, muſt be gauged by the following general

RULE.

To twice the Square of the Bung Diameter, add the Square of the Head-Diameter, and from this Sum take $\frac{4}{10}$ths (.4) of the Square of the Difference of the ſaid Diameters; multiply the Remainder by the Length of the Caſk, and divide the Product by 1077 for Ale, or by 882 for Wine, and the Quotient will give the Content required.

OPERATION.

$$\begin{array}{rr} 32 & 24 \\ 32 & 24 \\ \hline 64 & 96 \\ 96 & 48 \\ \hline 1024 & 576 \\ 2 & \\ \hline \end{array}$$

Twice the Sq. of the B. Diam.	2048
Square of the Head-Diameter	576
	2624
Sq. of 8, the Dif. of the Diam. multiplied by .4	=25.6
Difference	2598.4
Multiplied by the Length	42
	51968
	103936

1077)109132.8(101.32= the Content in Ale Gallons.

And by dividing by 882 (*viz.* three times 294), the Quotient will be 123.7, the Content in Wine Gallons.

Example 4.

Wherein it is propoſed to find the Content of a Caſk, in Wine Gallons, having the ſame Dimenſions (*i. e.* Bung and Head-Diameters, and Length) as that given in the 2d Example; only ſuppoſe the Diſtance *mn* to meaſure 3.3 Inches.

In this Caſe, the perpendicular Diſtance *mn* (ſee the preceding *Fig.*) being, *very nearly*, equal to $\frac{1}{8}$th of (26) the Difference of the Bung and Head-Diameters; this Caſk muſt therefore be gauged as the Middle Fruſtum of a *Parabolic Spindle*, by the laſt-mentioned general Rule.

Operation.

The Sq. of 65, the Bung-Diam. is 4225

The Double of which is 8450
The Sq. of 39, the Head-Diam. is 1521

Sum 9971
The Sq. of 26, the Dif. of Diam. mult. by .4 (*i.e.* 676 mult. by .4) = 270.4

Difference 9700.6
Multiplied by the Length 110

970060
97006

882)1067066.0(1209.82
W. Gall.

The Content of this Caſk will, by dividing by 1077, come out equal to 990.77 Ale Gallons.

As no general practical Rule* can poſſibly be given for finding, accurately, the Content of a Caſk compoſed of the Middle Fruſtum of an *Hyperbolic Spindle*, which we here denominate the *3d Variety*; therefore the beſt Way, when ſuch Caſks do occur, will be to have Recourſe to the following *general Rule*, which was firſt, and very judiciouſly, introduced into the preſent Subject, by the ingenious Mr. *Robert Shirtcliffe*: This Method evidently

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* If it was poſſible to give as *general* and *practical* a Rule, for determining the Meaſure of the Middle Fruſtum of an *Hyperbolic Spindle*, as thoſe are for a *Spheroidical Caſk*, and *Parabolic Spindle*; then the Buſineſs of Caſk-Gauging might be juſtly ſaid to be univerſally complete; ſince the Meaſure of the Fruſtum of the hyperbolic Spindle, will be ever ſome Quantity leſs than the *Fruſtum* of a parabolic Spindle, and greater than *that* of a Cone, the Diameters and Length being ſuppoſed the ſame in each Fruſtum.

For let A*m*BC*r*DA (*Fig.* XIII.) repreſent the Fruſtum of a parabolic Spindle, and A*n*BC*r*DA that of a Cone; let BE $= \frac{1}{2}$ the Difference of the Diameters, and from F, the Middle of AE, draw the Perpendicular F*m*; alſo draw *n e* and *mc*, each, parallel to AE: Then (by *Lemma* 1) B*c* $= \frac{1}{4}$ BE; but B*e* $= \frac{1}{2}$ BE; therefore (by *Lemma* 3) an Ordinate drawn from the Point of Interſection of the Diameter *mn*, and any hyperbolic Curve, paſſing through A and the Vertex B, may fall any-where between the Points *c* and *e*; conſequently the hyperbolic Curve may continually be varied, 'till it becomes coincident either with the parabolic Curve A*m*B, or the Right-line A*n*B; and that too, without varying the Diameters AB and BC, and the Length AE, of the aforeſaid Fruſtums; or, which is the ſame Thing, the Abſciſſa BE, and Ordinates AE and *n e*, of the hyperbolic Curve.

By the very ſame Method of Reaſoning (ſuppoſing BG drawn parallel to AE, cutting *nm* produced in *v*) it will appear (by *Lemma* 2) that the elliptic Curve, paſſing from A to the Vertex B, cannot deſcend ſo *low* as the Point *m*, nor yet riſe ſo *high* as *v*; conſequently no cloſe Caſk whatever (where the Vertex of the Curve is poſited in the Middle of the Caſk) can ſcarce contain more than the Middle Fruſtum of an *Elliptic Spindle*, nor leſs than *that* of an *Hyperbolic Spindle*, under the ſame Dimenſions, *i. e.* Head, Bung, and Length.

COROLLARY.

Hence it appears that two Caſks, compoſed of the Middle Fruſtums of hyperbolic Spindles, may have their correſponding Dimenſions equal, (*i. e.* the Head, Bung, and Length) and yet the Contents of theſe Caſks may greatly differ: The ſame is to be obſerved in the elliptic Spindle; but this Caſk (as before noticed) ſcarce ever occurs in Practice.

evidently follows from that of equidiſtant Ordinates (explained farther on).

Note. I ſhall hereafter exhibit a Table of Multipliers, for reducing a Caſk, of this Form of Curvature, into a Cylinder; theſe Multipliers are adapted to a Caſk of ſuch a Degree of Curvature, as I found (from repeated Experiments) would moſt frequently happen in this Variety.

THE GENERAL PROPOSITION.

Having the Length, Bung and Head-Diameters of a Caſk given, and alſo another Diameter taken exactly *in the Middle between the Bung and Head; to find the Content of the Caſk in Ale and Wine Gallons.*

THE GENERAL RULE.

To the Square of the Bung-Diameter, add the Square of the Head-Diameter, and alſo four times the Square of the Diameter taken *exactly* in the Middle between the Bung and Head; the Sum of theſe multiplied by the Length of the Caſk, and the Product divided by 2154.32 for Ale, or by 1764.7 for Wine, the Quotient will be the required Content of the Caſk.*

Note. The Middle Diameter is eaſily found, by ſubtracting twice the perpendicular Diſtance *mn* (ſee the *Fig. Pa.* 160) from the Bung-Diameter.

EXAMPLE

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* Or, univerſally, let the Solid be of what Form ſoever: Add the two extreme Areas, and four times that in the Middle together; multiply the Sum by one-ſixth of the Diſtance of the extreme Areas, and the Product will be the Meaſure of the Solid, *nearly*.

EXAMPLE 5.

To find the Content of a Caſk in Wine Gallons, having the ſame Bung and Head-Diameters, and Length, as that propoſed in the 2d Example; only let the Diſtance *mn*, be here ſuppoſed equal to 4.55 Inches.

The Diſtance *mn* (ſee *Fig. Pa.* 160) being, in this Caſe, more than $\frac{1}{6}$th of (26) the Difference of the Diameters; this Caſk, therefore, having the ſame Property as that compoſed of the Middle Fruſtum of an *Hyperbolic Spindle*, muſt be gauged by the preceding general Rule.

OPERATION.

Bung-Diameter 65
Twice the Diſtance *mn* (4.55) = 9.1

The Diam. in the Middle between the Bung and Head . . . . . } = 55.9

The Square whereof is 3124.81
Multiplied by 4

Gives 12499.24

The Sq. of 65, the B. Diam. is 4225
The Sq. of 39, the H. Diam. is 1521
Add 12499.24

Sum 18245.24
Multiplied by the Length . . . 110

18245240
1824524

1764.7(2006976.40(1137.29
= the Content in Wine Gallons.

By comparing the preceding Content, with that found for a Caſk of the ſame Dimenſions, in Example the 4th, it will appear that a very conſiderable Error may ariſe from computing the Content of this (or any other) Caſk, by the uſual Methods of only *gueſſing* at its Variety:—It is, indeed, very certain, that this laſt mentioned *general Rule* will give *very nearly* (and ſometimes *accurately)* the Content of any Caſk, let its Form be what it will; and the nearer the Head-Diameter approaches to an Equality with the Bung-Diameter, the leſs will be the Error.—But as we now have a general practical Method, for diſtinguiſhing the three different Varieties of Caſks; the Contents of the two firſt may therefore be found with greater Expedition, by the Rules given for thoſe two Varieties; ſee *Pages* 161 and 164: But, with Reſpect to the 3d Variety, the laſt-mentioned *general Rule (Pa.* 167) is preferable to all that have hitherto been, or, perhaps, ever can be propoſed.

EXAMPLE 6.

Wherein it is propoſed to find the Content of a Caſk in Wine Gallons, whoſe Bung-Diameter is 31, Head-Diameter 23, Length 50 Inches, and the Diſtance *mn* 1.4 Inches. (See *Fig. Pa.* 157.)

The Diſtance *mn* being greater than $\frac{1}{8}$th of (8) the Difference of the Bung and Head-Diameters; this Caſk, therefore, being of the *3d Variety*, muſt be gauged by the laſt-mentioned *general Rule.*

OPERATION.

Bung-Diameter 31
31

31
93

The Sq. of the B. Diam. 961
The Sq. of 23, the H. Diam. 529

Sum 1490

Bung Diameter 31
Twcie 1.4 (the Diſtance *mn*) = 2.8

The Diameter in the Middle between the Bung and Head } 28.2
28.2

564
2256
564

The Square of which is 795.24
Multiplied by . . . 4

Gives 3180.96
Add 1490.

Sum 4670.96
Multiplied by the Length 50

1764.7)233548.00(132.28
W. Gall.

Theſe Examples, I apprehend, will be ſufficient to enable the Learner to find, by the foregoing Method and Rules, the *Variety* and *Content* of a Caſk of any other Dimenſions.

The Contents of Caſks may be truly, and more expeditiouſly obtained, by firſt finding a *Mean-Diameter*; as will be fully explained in the next Section: But, before we enter upon that, it may not be amiſs to give two *general Rules*, for determining the Diameters taken in the *Middle*, between the Bung and Head-Diameters, of the ſpheroidical Caſk; and alſo that compoſed of the Middle Fruſtum of the parabolic Spindle.

PROP. I.

The Bung and Head-Diameters, and the Length of any Spheroidical Caſk *being given; to find the Diameter* exactly *in the Middle, between the Bung and Head.*

RULE.

To three times the Square of the Bung Diameter, add the Square of the Head-Diameter; $\frac{1}{4}$th of that Sum will be the Square of a Diameter in the *Middle* between the Bung and Head.*

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* Let the Bung and Head-Diameters, and $\frac{1}{2}$ the Length of any ſpheroidical Caſk, be repreſented by b, b, and d reſpectively, alſo let n repreſent any Diſtance in the Axis (leſs than d) from the Bung-Diameter: Then the Square of the ſemi-tranſverſe Axis of the whole *Spheroid*, being (by the Property of the Curve) expreſſed by $\frac{b^2d^2}{b^2-b^2}$, we have (again by the Property of the Curve) $\frac{b^2d^2}{b^2-b^2} : b^2$ (or $\frac{d^2}{b^2-b^2} : 1$) $:: \overline{\frac{bd}{\sqrt{b^2-b^2}}+n} \times \overline{\frac{bd}{\sqrt{b^2-b^2}}-n}$: the Square of the Diameter, at n Diſtance from the Bung-Circle; which is therefore expreſſed by $\frac{b^2-b^2}{d^2} \times \overline{\frac{b^2d^2}{b^2-b^2}-n^2}$, or $b^2-\frac{n^2\times\overline{b^2-b^2}}{d^2}$, Which, when $n=\frac{d}{2}$ (as in the above *Prop.*), becomes $b^2-\frac{b^2-b^2}{4}$ ($=\frac{3b^2+b^2}{4}$) $=$ the Square of the Diameter in the *Middle* between thoſe of the Bung and Head. Q. E. I.

PROP. II.

The Bung and Head-Diameters, of the Middle Fruſtum of a Parabolic Spindle *being given; to find the Diameter in the* Middle *between the Bung and Head.*

RULE.

From the Bung-Diameter, ſubtract $\frac{1}{4}$th of the Difference of the Bung and Head-Diameters, the Remainder will be the Diameter in the Middle between the Bung and Head.†

EXAMPLE I.

Suppoſe a *Spheroidical Caſk*, whereof the Bung-Diameter is 32, Head-Diameter 24, and the Length 42 Inches; it is required to find the Diameter in the Middle between the Bung and Head: And alſo the Content of the Caſk in Ale Gallons, by the *general Rule*, *Pa.* 167.

First, by the Rule preceding the laſt, we have three times the Square of the Bung Diameter = 3072
The Square of 24, the Head Diam. = 576

Sum is 3648

$\frac{1}{4}$th is 912, the Square of the Middle Diameter.

Then, by the *general Rule* (*Pa.* 167) we have the following

OPERATION.

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† This Rule is very evident from the Property of the *Parabola*, ſee *Lemma* 1, *Pa.* 158.

OPERATION.

The Sq. of 32, the Bung Diam. = 1024
The Sq. of 24, the Head Diam. = 576
Four times 912, the Square of the Middle Diameter } = 3648

Sum 5248
Multiplied by the Length . . 42

10496
20992

2154.32)220416(102.32 = the Content in Ale Gallons, *exactly* agreeing with *that* found by the common Method (ſee *Example* 1, *Pa.* 161).

EXAMPLE 2.

Wherein it is required to find the *Middle-Diameter*; and alſo the Content in Ale Gallons (by the *general Rule*, *Pa.* 167) of a Caſk of the 2d Variety, whoſe Bung and Head-Diameters, and Length, are the ſame as in the preceding Example.

OPERATION.

The Difference of the Diameters 8

$\frac{1}{4}$th is 2, which being ſubtracted from 32, the Bung-Diameter, leaves 30, the Middle Diameter; agreeable to the *preceding Rule*.

Then the Sq. of 32, the B. Diam. is 1024
The Square of 24, the H. Diam. is 576
Four times the Square of 30, the Middle-Diameter, is } 3600

Sum 5200
Multiplied by the Length . . 42

10400
20800

2154.32)218400(101.37

= the Content in Ale Gallons, *very nearly* agreeing with that found by the common Method, in Example 3d, *Pa.* 163.

The two laſt Examples were only given *here*, to ſhew the Conformity between the *general Propoſition (Pa.* 167) and the common Method of finding the Contents of theſe two Caſks; which, in the firſt Variety, will always be an *exact* Agreement; ‡ and

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‡ The Reaſon of the *general Rule (Pa.* 167) bringing out *preciſely* the Meaſure of the ſpheroidical Caſk, is from hence:—If b, b, M, and d, denote the Bung, Head, Middle Diameter, and Length reſpectively, of any ſpheroidical Caſk, and $p = .7854$; then inſtead of M^2, in the Expreſſion $(\frac{pd}{6} \times \overline{b^2 + 4M^2 + b^2})$ for the Content, ſubſtitute its Equal $\frac{3b^2 + b^2}{4}$, found from the Property of the Curve, *Pa.* 171, and we ſhall get $\frac{pd}{6} \times \overline{b^2 + 3b^2 + b^2 + b^2}$ (or $\frac{pd}{3} \times \overline{2b^2 + b^2}$), which is well known to be the *accurate Meaſure* of every ſpheroidical Caſk.

COROLLARY.

Hence we can eaſily determine when the Anſwer brought out by the *general Rule (Pa.* 167) is *ſtrictly true*; provided we have another *Rule*, or Method, whereby the *true* Meaſure of a Plane of a known Form, or a Solid generated by the Revolution of a Curve of a known Property, can be found: For if inſtead of (M) the Middle Perpendicular (if a Surface), or (M^2) its Square (if a Solid) we ſubſtitute its Equal, found from the Property of the Curve (or and in the ſecond, the Difference will be inconſiderable in Practice.

(or Figure); then, if there reſults the known *accurate* Rule for determining the Meaſure of the Figure, it is evident (in ſuch Caſe) the *general Rule* hereafter given for three equidiſtant Perpendiculars, and alſo that given (*Pa.* 167) for three equidiſtant perpendicular Planes, will be *ſtrictly true*.

Thus, for Inſtance, in the Fruſtum of a ſquare Pyramid, if a^2 and b^2 denote the Areas of the two Ends, and d the Altitude of the ſaid Fruſtum; then, in the general Expreſſion $\overline{a^2+4M^2+b^2}\times\frac{d}{6}$, for M^2 ſubſtitute its Equal $\left.\overline{\frac{a+b}{2}}\right|^2$, found from the Property of the Figure, and we ſhall have $\overline{a^2+a^2+2ab+b^2+b^2}\times\frac{d}{6}$, or $\overline{a^2+ab+b^2}\times\frac{d}{3}$, which is known to to be the *accurate* Meaſure of the Fruſtum. Q. E. I.

OTHERWISE, *let the Fruſtum of the Pyramid be what it will.*

Let any two homologous Sides of the greater and leſſer ſimilar Ends of the Fruſtum, be denoted by a and b reſpectively; and let a correſponding Side of a parallel Section in the Middle be denoted by M: Then will a^2, b^2, and M^2, be as the Meaſures of the three parallel Planes reſpectively; and therefore, by the *general Rule*, *Pa.* 167, $\overline{a^2+4M^2+b^2}\times\frac{d}{6}$ will be as the Content of the Fruſtum (d repreſenting the perpendicular Diſtance of the two extreme Planes): — But $\frac{a+b}{2}=$ the correſponding Side of the Plane in the Middle; $\therefore \left.\overline{\frac{a+b}{2}}\right|^2=M^2$, which being ſubſtituted above, in the general Expreſſion for the Content, we get $\overline{a^2+a^2+2ab+b^2+b^2}\times\frac{d}{6}$ or $\overline{a^2+ab+b^2}\times\frac{d}{3}$, which is well known to be as the *accurate* Meaſure of the Fruſtum of any Pyramid (or Cone) whatever. Vid. *Sect.* VIII. *Pa.* 114.

SECTION

SECTION X.

OF FINDING THE MEAN-DIAMETERS OF CASKS.

THERE are two Methods, now in Practice, for finding the Mean-Diameter of a Caſk, or reducing it to a Cylinder, of the ſame Length and Magnitude; the firſt is, by multiplying the Difference of the Bung and Head-Diameters, by ſome conſtant, or fixed, Multiplier (as by .7 for a ſpheroidical Caſk, .68 for the Middle Fruſtum of a parabolic Spindle, *&c.* according to the Variety of the Caſk), and adding that Product to the Head-Diameter, this Sum is called the *Mean-Diameter* of the Caſk; which is erroneous, as will be ſhewn hereafter.

The other Method is, by the Tables which are to be found in moſt Authors on Gauging, and are alſo graduated on one Edge of the Sliding-Rule; but, it is plain, thoſe Tables are formed from a Conſideration that *all* Caſks which have the ſame Difference between the Bung and Head-Diameters, muſt likewiſe have one conſtant Multiplier; therefore this Method is alſo defective: — For it is abſolutely impoſſible there ſhould be any *conſtant Multiplier*, uſed in reducing Caſks (even of the ſame Variety) to Cylinders, of the ſame Lengths and Magnitudes with thoſe Caſks; unleſs it be ſuch which have the Bung and Head-Diameters in ſome conſtant Proportion; for the *Multipliers* muſt vary, when theſe Proportions vary, as will be hereafter made to appear: It may ſuffice to ſhew here, by an eaſy Example, that two Casks (of the ſame Variety) may have the very ſame Difference of their Bung and Head-Diameters, and yet the Proportion of the Diameters of each Cask, may be very different.

Thus, let 32 and 24 Inches be the Bung and Head-Diameters of one Caſk, and thoſe of another be 48 and 40 Inches: Here the Difference of the Diameters is the ſame in each Cask, but the Proportion of their Diameters is unlike; for, in the firſt Caſe, the Bung-Diameter contains the Head and $\frac{1}{3}$d Part thereof; but in the latter, the greater exceeds the leſſer by $\frac{1}{5}$th Part *only*: Theſe two Casks therefore, though the Difference of the Diameters is the ſame in both, require different Multipliers.

From the Inveſtigations* in the ſubjoined Notes, two different Methods may be given for finding a

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* Let the Head-Diameter of any ſpheroidical Caſk be called x, the Bung-Diameter y, and let m denote ſome Multiplier, by which the ſaid Bung-Diameter being multiplied, the Product will give the *Mean-Diameter* of the Caſk; or, which is the ſame Thing, the Diameter of a Cylinder, whoſe Length and Magnitude are equal to *thoſe* of any propoſed ſpheroidical Caſk: Hence (by the well-known Theorem) we get $\frac{2y^2+x^2}{3}=m^2y^2$; therefore $x=\sqrt{3m^2y^2-2y^2}=y\sqrt{3m^2-2}$; conſequently the Bung-Diameter is to the Head-Diameter, *univerſally*, as $y : y\sqrt{3m^2-2}$; or as $1 : \sqrt{3m^2-2}$: From whence it is eaſy to perceive, that, when the Multiplier (m) is varied, the Ratio of the Bung and Head-Diameters muſt vary; conſequently there cannot be any conſtant, or fixed, Multiplier.

Moreover it evidently appears, that the Multiplier (m) cannot be greater than Unity, nor leſs than $\sqrt{\frac{2}{3}}$; therefore all the Multipliers, or Values of m, let the Bung and Head-Diameters be what they will, are included between 1 and .8164 &c. (*viz.* $\sqrt{\frac{2}{3}}$): — But it will be unneceſſary to extend any of the Tables, to contain Multipliers for a Caſk (or Veſſel) whoſe Bung (or greater) Diameter, is more than twice the Head (or leſs) Diameter.

Mean-Diameter, and both of them equally accurate and comprehenſive; the firſt is, by multiplying the Bung-Diameter by a Number, or Factor, according to the Proportion of the Bung and Head-Diameters, this Product will give the *Mean-Diameter*; the other Method is, by multiplying the Difference of the Diameters by a Number, or Factor, which muſt alſo be according to the Proportion of the two Diameters of the Caſk or Veſſel; and that Product being added to the leſs Diameter, the Sum will be the *Mean-Diameter*.

But, as it might be deemed unneceſſary to exemplify both theſe Methods, I thought it would ſuffice to only put down for the firſt, the Tables of Multipliers, as they are derived from a different Conſideration than any hitherto offered to the Public; but for the other Method, I have given both Tables of Factors, and proper Examples to illuſtrate the ſame.

By theſe laſt Tables it will plainly appear, that the common Factors .7, .68, *&c.* uſed in reducing Caſks to Cylinders (notwithſtanding they are better adapted to Practice than any other conſtant Factors whatever), are only *ſtrictly true* in particular Circumſtances: And though the ſaid Factors will be ſufficiently near the Truth, in finding the Contents of many Caſks which occur in Practice; yet, it is very certain, when the Caſk is ſomewhat out of the common Form, the Error will then be far too conſiderable to be diſregarded; ſo that I preſume theſe Tables will be found of great Utility, in determining the *true* Content of a Veſſel in any of the following Forms; namely, for a cloſe

Caſk, either in the Form of the Middle Fruſtum of a *Spheroid*, *Parabolic Spindle*,* or *Hyperbolic Spindle*;

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* Let the Head-Diameter of a Caſk, repreſenting the Middle Fruſtum of a parabolic Spindle, be denoted by x, and the Bung-Diameter by y, and let (as in the preceding *Note)* the variable Multiplier be called m; then (by the Writers on Fluxions) we have $\frac{8y^2+3x^2+4yx}{15}=m^2y^2$, or $\frac{3x^2}{15}+\frac{4yx}{15}=m^2y^2-\frac{8y^2}{15}$; $\therefore x^2+\frac{4yx}{3}=5m^2y^2-\frac{8y^2}{3}$; whence $x=\sqrt{5m^2y^2-\frac{20y^2}{9}}-\frac{2y}{3}=y\times\sqrt{5m^2-\frac{20}{9}}-\frac{2}{3}$; conſequently the Bung-Diameter is to the Head-Diameter, *univerſally*, as $y:y\times\sqrt{5m^2-\frac{20}{9}}-\frac{2}{3}$, or as $1:\sqrt{5m^2-\frac{20}{9}}-\frac{2}{3}$: Hence it appears, that, when the Ratio of the Bung and Head-Diameters varies, the Multiplier *(m)* muſt vary: Moreover it is evident, that the Multiplier *(m)* cannot be greater than *Unity*, nor leſs than $\sqrt{\frac{8}{15}}$, *viz*. .7302, &c.

† Let x be the leſs, and y the greater Diameter either of the Fruſtum of a parabolic Conoid or that of a Cone, alſo let m be a Multiplier, by which if the greater Diameter be multiplied, the Product ſhall be the Mean-Diameter; hence (by the well-known Theorems) we have the following Equations:

Viz. For the Fruſtum of a *Parabolic Conoid*, $\frac{y^2+x^2}{2}=m^2y^2$;

For the Fruſtum of a *Cone*, $\frac{y^2+yx+x^2}{3}=m^2y^2$.

These two Equations, being ſolved, give $x=\begin{cases} y\sqrt{2m^2-1}, & \text{for the Fruſtum of a } \textit{Parabolic Conoid}. \\ y\sqrt{3m^2-\frac{3}{4}}-\frac{1}{2}y, & \text{for the Fruſtum of a } \textit{Cone}. \end{cases}$

Therefore the greater Diameter is to the leſs, *univerſally*,

As $\begin{cases} 1:\sqrt{2m^2-1}, & \text{for the Fruſtum of a } \textit{Parabolic Conoid}. \\ 1:\sqrt{3m^2-\frac{3}{4}}-\frac{1}{2}, & \text{for the Fruſtum of a } \textit{Cone}. \end{cases}$

Hence it is evident, that when the Ratio of the two Diameters of each Fruſtum varies, the Multiplier *(m)* muſt vary: It is likewiſe evident, that the ſaid Multiplier, in the firſt Caſe, cannot be greater than Unity, nor leſs than $\sqrt{\frac{1}{2}}$, or .7071, &c. and in the Fruſtum of a Cone, the Multiplier *(m)* cannot exceed Unity, nor be leſs than $\overline{\frac{1}{3}}$, *viz*. .5773, &c.

By

Spindle;‡ and alſo, for an open Utenſil, the Fruſtum of a *Parabolic Conoid* and *Cone*.†

And

By the foregoing Proportions it appears, that the Limits of the Multipliers, for the Bung (or greater) Diameters, are

$$\text{Between}\left\{\begin{array}{l} .8164, \text{\&c.} \left(\sqrt{\tfrac{2}{3}}\right) \\ .7302, \text{\&c.} \left(\sqrt{\tfrac{8}{13}}\right) \\ .7071, \text{\&c.} \left(\sqrt{\tfrac{1}{2}}\right) \\ .5773, \text{\&c.} \left(\sqrt{\tfrac{1}{3}}\right) \end{array}\right\}$$

and Unity: Moreover it is evident (from the Method hereafter ſhewn of deriving Tables 3 and 4, from 1 and 2 reſpectively) that the Limits for the Multipliers, for the Difference of the Diameters, are

$$\text{Between}\left\{\begin{array}{ll} .6666, \text{\&c.} & \left(\tfrac{2}{3}\right) \\ .6666, \text{\&c.} & \left(\tfrac{2}{3}\right) \\ .5 & \left(\tfrac{1}{2}\right) \\ .5 & \left(\tfrac{1}{2}\right) \end{array}\right\} \text{and} \left\{\begin{array}{l} .8164, \text{\&c.} \\ .7302, \text{\&c.} \\ .7071, \text{\&c.} \\ .5773, \text{\&c.} \end{array}\right.$$

The *Multipliers* (*Table* I.) for ſpheroidical Caſks, and for the Middle Fruſtum of parabolic Spindles; likewiſe *thoſe* (*Table* II.) for the Fruſtums of parabolic Conoids and Cones, were derived from the *general Proportions* of the Bung and Head, or greater and leſs Diameters (ſee *Pa.* 177 and 179). — For, by aſſuming the Ratio of the two Diameters, we can readily obtain the Value of *m*: Thus, for Inſtance, let the Ratio of the greater and leſs Diameters be as 2 to 1; then we have

$$\left.\begin{array}{ll} \text{For the Middle Fruſtum of a } \textit{Spheroid}, & 1 : \sqrt{3m^2-2} \\ \text{For the Middle Fruſtum of a } \textit{Parabolic Spindle} \ldots & 1 : \sqrt{5m^2-\frac{20}{9}}-\frac{2}{3} \\ \text{For the Fruſtum of a } \textit{Parabolic Conoid}, & 1 : \sqrt{2m^2-1} \\ \text{For the Fruſtum of a } \textit{Cone} \ldots\ldots & \sqrt{3m^2-\frac{3}{4}}-\frac{1}{2} \end{array}\right\} :: 2 : 1 \; (:: 1 : 0.5).$$

Whence we get for the

$$\left.\begin{array}{l} \text{1ſt} \\ \text{2d} \\ \text{3d} \\ \text{4th} \end{array}\right\} m = \left\{\begin{array}{l} .866 \\ .8464 \\ .7905 \\ .7637 \end{array}\right\} = \text{the } \textit{general Multipliers} \text{ for the Bung (or greater)}$$

Diameter of any Caſk, or Veſſel, in the above Forms, whoſe Diameters are in the Ratio of 2 to 1: — Or if the Bung (or greater) Diameter be expreſſed by Unity, and the Head (or leſs) Diameter by $\frac{1}{2}$; then the above Numbers will expreſs the Mean-Diameters themſelves.

‡ The Multipliers for the Middle Fruſtums of the *Hyperbolic Spindle* (*Tab.* I.) were derived in the following Manner.

It was firſt found (by various Experiments) that many Caſks, whoſe Contents were leſs than thoſe of *Parabolic Spindles* (having the ſame Bung, Head, and Length) had the Difference of the Bung and Head-Diameters, and the Diſtance *mn* (ſee *Fig. Pa.* 157 or 160) in the Ratio of 8 : 1.4.

Now if b, b, and M, denote the Bung, Head, and Middle Diameters reſpectively of any Caſk; and alſo m the Mean-Diameter thereof; then, by the

And I flatter myſelf, that the Advantage of the following Tables will be acknowledged by the *attentive* and *unprejudiced* Reader, as better adapted to *real* Practice, than any hitherto publiſhed; conſidering both the Facility of the Operations, and the Accuracy of the Concluſions: For, by the Method here laid down, the Contents of cloſe Caſks (and open

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the *general Propoſition*, *Pa.* 167, we ſhall have $\frac{b^2+b^2+4M^2}{6}=m^2$, $\therefore m = \sqrt{\frac{b^2+b^2+4M^2}{6}}$: Whence it is evident that if the Bung-Diameter be denoted by Unity, and the Head-Diameter by any Number leſs than Unity, ſuppoſe, for Example, by .75, we ſhall have $8 : 1.4 :: .25$ (1.75) : $\frac{1.4\times.25}{8}=.04375$ ($= mn$, ſee *Fig. Pa.* 160), the Double whereof is .0875; then $1-.0875=.9125=$ the Middle Diameter; therefore, in this Caſe, $b=1$, $b=.75$, and $M=.9125$; conſequently $\sqrt{\frac{b^2+b^2+4M^2}{6}}$ $(=m)=\sqrt{\frac{1^2+.75^2+4\times.9125^2}{6}}=.903=$ the Mean-Diameter, or *general Multiplier*, for the Middle Fruſtum of an *Hyperbolic Spindle* (of this Form) whoſe Head-Diameter is equal to ¾ of the Bung-Diameter.

The 3d and 4th Tables, are reſpectively deduced from the 1ſt and 2d; in the following Manner.

Having already proved, that the Multiplier (m) depends intirely upon the Ratio of the two Diameters of the Fruſtum; and therefore, in the 1ſt and 2d Tables, the Bung (or greater) Diameter being denoted by Unity, we have the Head (or leſs) Diameter expreſſed in Decimal Parts, in the Columns titled *the Quotient of the Head (or leſs) Diameter, divided by the Bung (or greater) Diameter*; and in the other Columns (titled *Multipliers, &c.*) ſtand the *true* Mean-Diameters for the reſpective Caſk, *&c.* whoſe Bung and Head (or greater and leſs Diameters) are as here ſpecified: Thus, for Example, call the Bung-Diameter 1, the Head-Diameter .6, then the Mean-Diameter (or Multiplier) of ſuch a *Spheroidical Caſk* (by *Tab.* I.) is .887; whence (by multiplying the Difference of the Diameters by x, and adding the Product to the Head-Diameter) we get $.4x+.6=.887$, $\therefore x=\frac{.287}{.4}=.7175$ for a Multiplier; whereby the Difference of the Diameters of every ſpheroidical Caſk, having the Bung and Head-Diameters in the Ratio of 5 to 3, muſt be multiplied, and the Product added to the Head-Diameter, in Order to obtain the Mean-Diameter: The ſame Method muſt be obſerved in finding the Multipliers for the other Varieties.

open Veſſels) may, with the utmoſt Exactneſs, be as expeditiouſly obtained, as by that *uncertain* Method of uſing the fixed Multipliers.

TABLE I.

Exhibiting the Multipliers, *whereby if the Bung-Diameters of Caſks, reſembling the Middle Fruſtums of* Spheroids, *or of* Parabolic *and* Hyperbolic Spindles, *be multiplied; the Products will give the* Mean-Diameters *thereof.*

Quotient of the H. Diam. divided by the B. Diam.	Multipliers for *Spheroidical Caſks.*	Multipliers for *the Fruſtums of Parabolic Spindles.*	Multipliers for the *Fruſtums of an Hyperbolic Spindle.*
.50	.866	.8465	.8136
.51	.868	.8493	.817
.52	.87	.852	.8204
.53	.872	.8548	.8239
.54	.874	.8576	.8273
.55	.876	.8605	.8308
.56	.878	.8633	.8343
.57	.8802	.8662	.8378
.58	.8824	.869	.8413
.59	.8846	.872	.8448
.60	.887	.8748	.8483
.61	.8892	.8777	.8518
.62	.8915	.8806	.8554
.63	.8938	.8835	.859
.64	.8962	.8865	.8626
.65	.8986	.8894	.8662
.66	.901	.8924	.8698
.67	.9034	.8954	.8735
.68	.906	.8983	.8772
.69	.9084	.9013	.8808
.70	.911	.9044	.8845
.71	.9136	.9074	.8881
.72	.9162	.9104	.8918
.73	.9188	.9135	.8955
.74	.9215	.9166	.8993
.75	.9242	.9196	.903
.76	.927	.9227	.9068
.77	.9296	.9258	.9105
.78	.9324	.929	.9143
.79	.9352	.932	.9181
.80	.9388	.9352	.9219
.81	.9409	.9383	.9257
.82	.9438	.9415	.9295
.83	.9467	.9446	.9333
.84	.9496	.9478	.9372
.85	.9526	.951	.941
.86	.9556	.9542	.9449
.87	.9586	.9574	.9487
.88	.9616	.9605	.9526
.89	.9647	.9638	.9565
.90	.9678	.961	.9604
.91	.971	.9703	.9643
.92	.974	.9736	.9682
.93	.9772	.9768	.9722
.94	.9804	.9801	.9761
.95	.9836	.9834	.9801
.96	.9868	.9867	.9841
.97	.99	.99	.988
.98	.9933	.9933	.992
.99	.9966	.9966	.996
1.00	1.0000	1.0000	1.000

TABLE

TABLE II.

Shewing the Multipliers, *by which if the greater Diameters of the Fruſtums of* Parabolic Conoids *and* Cones, *be multiplied; the Products will give the* Mean-Diameters *thereof.*

Quot. of the leſſer Diam. divided by the greater Diam.	Multipliers for the *Fruſtums of Parabolic Conoids.*	Multipliers for the *Fruſtums of Cones.*	*Quot. of the leſſer Diam. divided by the greater Diam.*	Multipliers for the *Fruſtums of Parabolic Conoids.*	Multipliers for the *Fruſtums of Cones.*
.50	.7905	.7637	.76	.8881	.8827
.51	.7937	.7681	.77	.8924	.8874
.52	.797	.7725	.78	.8967	.8922
.53	.8002	.7769	.79	.9011	.897
.54	.8036	.7813	.80	.9055	.9018
.55	.807	.7858	.81	.91	.9066
.56	.8104	.7902	.82	.9144	.9114
.57	.814	.7947	.83	.9188	.9163
.58	.8174	.7992	.84	.9234	.9211
.59	.821	.8037	.85	.928	.926
.60	.8246	.8082	.86	.9326	.9308
.61	.8282	.8128	.87	.9372	.9357
.62	.832	.8173	.88	.9419	.9406
.63	.8357	.822	.89	.9466	.9455
.64	.8395	.8265	.90	.9513	.9504
.65	.8433	.8311	.91	.956	.9553
.66	.8472	.8357	.92	.9608	.9602
.67	.8511	.8404	.93	.9656	.9652
.68	.8551	.845	.94	.9704	.9701
.69	.859	.85	.95	.9753	.975
.70	.8631	.8544	.96	.9802	.98
.71	.8672	.859	.97	.9851	.985
.72	.8713	.8637	.98	.99	.99
.73	.8754	.8685	.99	.995	.995
.74	.8796	.8732	1.00	1.000	1.000
.75	.8838	.878			

By the preceding Tables, the *Mean-Diameters* of Caſks, compoſed of the Middle Fruſtums of Spheroids, and of parabolic and hyperbolic Spindles; and likewiſe the *Mean-Diameters* of the Fruſtums of parabolic Conoids and Cones, may very readily be obtained, by the following general

RULE.

Divide the Head (or leſs) Diameter, by the Bung (or greater) Diameter, to two Places of Decimals in the Quotient, againſt which, in the proper Column, we have a *Decimal Fraction*; which being multiplied by the Bung (or greater) Diameter, the Product will give the *true* Mean-Diameter ſought.

TABLE

TABLE III.

Exhibiting the Multipliers, *whereby if the Difference of the Bung and Head-Diameters of a* Spheroidical *Caſk, or that composed of the Middle Fruſtum of a* Parabolic *or* Hyperbolic Spindle, *be multiplied, and the Product added to the Head Diameter; the Sum will give the* Mean-Diameter *thereof:* (i. e. *of any propoſed Caſk, within the Limits of this* Table.)

Quotient of the H. Diam. divided by the B. Diam.	Multipliers for *Spheroidical Caſks.*	Multipliers for *the Fruſtums of Parabolic Spindles.*	Multipliers for the *Fruſtum of an Hyperbolic Spindle.*	*Quotient of the H. Diam. divided by the B. Diam.*	Multipliers for *Spheroidical Caſks.*	Multipliers for *the Fruſtums of Parabolic Spindles.*	Multipliers for the *Fruſtum of an Hyperbolic Spindle.*
.50	.732	.693	.627	.76	.695	.678	.611
.51	.73	.692	.626	.77	.693	.677	.611
.52	.729	.691	.626	.78	.692	.677	.61
.53	.727	.691	.625	.79	.691	.676	.61
.54	.726	.69	.624	.80	.69	.676	.609
.55	.724	.689	.624	.81	.688	.675	.609
.56	.722	.689	.623	.82	.687	.675	.608
.57	.721	.688	.623	.83	.686	.674	.607
.58	.72	.688	.622	.84	.685	.673	.607
.59	.718	.687	.621	.85	.684	.673	.606
.60	.717	.687	.621	.86	.682	.672	.606
.61	.715	.686	.62	.87	.681	.672	.605
.62	.714	.686	.619	.88	.68	.671	.605
.63	.713	.685	.618	.89	.679	.671	.604
.64	.711	.684	.618	.90	.678	.671	.604
.65	.71	.684	.617	.91	.677	.67	.603
.66	.708	.683	.617	.92	.675	.67	.603
.67	.707	.683	.616	.93	.674	.668	.603
.68	.706	.682	.616	.94	.673	.668	.602
.69	.704	.681	.615	.95	.672	.668	.602
.70	.703	.681	.615	.96	.67	.667	.602
.71	.702	.681	.614	.97	.669	.667	.601
.72	.701	.68	.613	.98	.667	.666	.601
.73	.699	.679	.612	.99	.666	.666	.6&c.
.74	.698	.679	.612	1.00	—[&c.	—[&c.	—
.75	.697	.678	.612				

Note. *The above Table (for the Sake of Convenience) is now graduated on the Sliding-Rule, as made by that ingenious Mathematical-Inſtrument-Maker, Mr.* John Bennett, *in* Crown-Court, St. Ann's, Soho.

Two Places of Decimals being taken for a Multiplier, in the Manner as they are now placed on the Sliding-Rule, will give the Mean-Diameter of a Caſk to a ſurprizing Degree of Exactneſs: But I judged it would not be amiſs to give three Places in the preceding Table, in Order to ſhew in what Circumſtances (with Regard to the Proportion of the Bung and Head-Diameters) the common Multipliers (.7 and .68) will be the moſt exact.

I ſhall now proceed to ſhew the Utility of this laſt Table, by the following Examples.

GENERAL RULE.

Divide the Head-Diameter by the Bung-Diameter, to two Places of Decimals in the Quotient, againſt which, in the Column anſwering to the propoſed Variety, we have a Decimal; which being multiplied by the Difference of the Bung and Head-Diameters, and the Product being added to the Head-Diameter, the Sum thereof will be the *true* Mean-Diameter ſought.

EXAMPLE I.

Wherein it is propoſed to find the Mean-Diameter, and Content of a *Spheroidical Caſk*, in Wine Gallons; whereof the Bung-Diameter is 65, Head-Diameter 39, and the Length 110 Inches.

OPERATION.

65) 39.0 (.6 *Quotient.*
390

Then againſt .60 (*Tab.* III.) in the firſt Column, and in that for ſpheroidical Caſks, we have the
Multiplier (or Factor) . . . . . = .717
Multiplied by the Diff. of the Diam. = 26

4302
1434

Product 18.642
Head-Diameter 39

Mean-Diameter 57.642, the Area, in Wine Gallons, anſwering to this Diameter, is . . . . 11.302 *very near.*
Multiplied by the Length 110

113020
11302

Gives 1243.220 Wine Gallons, the Content of the Caſk; the ſame as was found in *Example* 2, ſee *Pa.* 163.

If, in the foregoing Example, the Difference of the Bung and Head-Diameters be multiplied by .6, agreeable to an Obſervation of a very celebrated Author, the Mean-Diameter will come out 54.6 Inches; and therefore the Content of the Caſk will then *appear* to be but 1114.3 Wine Gallons, which is 129 Gallons *leſs* than the *Truth!*

EXAMPLE 2.

Let it be propoſed to find the Mean-Diameter, and Content in Wine Gallons, of a Caſk repreſenting the Middle Fruſtum of a *Parabolic Spindle*; whoſe Bung-Diameter is 32, Head-Diameter 24, and the Length 42 Inches.

OPERATION.

32)24.00(.75 *Quotient.*

Then againſt .75 (*Tab.* III.), in the proper Column for this *Variety*, we have .678 for a Multiplier; and therefore, by proceeding as in the laſt Example, the Mean-Diameter is 29.424, and the required Content 123.648 Wine Gallons; the ſame as found by the general Rule, *Exam.* 3, *Pa.* 164.

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Length 42 on C; then againſt 29.4, the Mean-Diameter on D, we have 124 Gallons *nearly*, the Content of the Caſk on C.

EXAMPLE 3.

Suppoſe the Dimenſions of a Caſk of the 3d Variety (or the Middle Fruſtum of an *Hyperbolic Spindle*) be the ſame as were given in Example 2, *Pa.* 162; to find the Mean-Diameter of the Caſk, and its Content in Wine Gallons.

OPERATION,

Operation.

If the Head-Diameter 39, be divided by the Bung-Diameter 65, the Quotient will be .6; againſt which (*Tab.* III.), in the Column proper for this Variety, we have 621

Multiplied by the Difference of the Bung and Head-Diameters } 26

3726
1242

Product 16.146
Head-Diameter 39

Mean-Diameter is 55.146, the Area in Wine Gallons, anſwering to this Diameter, is 10.34, which being multiplied by the Length (110) gives 1137.4 Wine Gallons, the required Content of the Caſk: Which differs 106 Gallons from one of a ſpheroidical Form, having the ſame Bung, Head, and Length, ſee *Pa.* 163; but agrees, *very nearly*, with the Content found according to the *general Rule*, ſee *Example* 5, *Pa.* 168.

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Length 110 on the Line C (*i. e.* on the 1ſt Radius); then againſt the Mean-Diameter 55.14 on D, we have 1137.4 Gallons, the Content on C *as before.*

Example 4.

Wherein it is propoſed to find the Mean-Diameter, and Content of a Caſk of the 3d Variety (or the

Middle Fruſtum of an *Hyperbolic Spindle)*, whoſe Bung-Diameter is 31, Head-Diameter 23, and the Length 50 Inches.

By dividing the Head by the Bung-Diameter, and proceeding in the very ſame Manner as in the foregoing Examples, we ſhall find the Mean-Diameter 27.89, and the Content of the Caſk 132.25 Wine Gallons, the ſame as in *Pa.* 170.

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Length 50 on C, then againſt 27.19, the Mean-Diameter on D, we have 132.25 Wine Gallons, the Content on C, the ſame *as above.*

Note. If the above Example be wrought by the common Method, of uſing .7 for a Multiplier, the Content will then *appear* to be 139 Gallons, which exceeds the *true* Meaſure 6.75 Gallons.

TABLE

TABLE IV.

Shewing the Multipliers, *whereby the Difference of the Diameters of the Frustums of a* Parabolic Conoid *or* Cone, *being multiplied, and the Product added to the less Diameter; the Sum will give the* Mean-Diameter *thereof*: (i. e. *of any proposed Frustum, within the Limits of this* Table.)

Quotient of the less Diam. divided by the greater Diam.	*Multipliers for the Frustums of Parabolic Conoids.*	*Multipliers for the Frustums of Cones.*	*Quotient of the less Diam. divided by the greater Diam.*	*Multipliers for the Frustums of Parabolic Conoids.*	*Multipliers for the Frustums of Cones.*
·50	·581	·527	·76	·534	·511
·51	·579	·526	·77	·532	·51
·52	·577	·526	·78	·53	·51
·53	·575	·525	·79	·529	·51
·54	·573	·524	·80	·527	·509
·55	·571	·524	·81	·526	·508
·56	·569	·523	·82	·524	·507
·57	·567	·522	·83	·522	·507
·58	·565	·521	·84	·521	·507
·59	·563	·521	·85	·52	·506
·60	·562	·52	·86	·518	·506
·61	·56	·52	·87	·517	·505
·62	·558	·519	·88	·515	·505
·63	·556	·519	·89	·514	·504
·64	·554	·518	·90	·513	·504
·65	·552	·517	·91	·511	·503
·66	·55	·517	·92	·51	·502
·67	·549	·516	·93	·508	·502
·68	·547	·516	·94	·507	·501
·69	·545	·516	·95	·506	·501
·70	·543	·515	·96	·505	·5 &c.
·71	·541	·514	·97	·503	·5
·72	·54	·513	·98	·501	·5
·73	·538	·513	·99	·5 &c.	·5 &c.
·74	·537	·512	1.00	—	—
·75	·535	·512			

It appears from the foregoing Table, that the Mean-Diameter of the Fruſtum of a Cone is *nearly* equal to half the Sum of the top and bottom Diameters of the ſaid Fruſtum; eſpecially when the leſs Diameter is more than $\frac{2}{3}$ds of the greater Diameter: But this Obſervation, it is evident from Table III. will not hold good, with Reſpect to the other Fruſtums, in any Circumſtance whatever.

EXAMPLE 5.

The greater Diameter of the Fruſtum of a *Parabolic Conoid* is 45, the leſs Diameter 27, and the Altitude 40 Inches; required the Mean-Diameter of the Fruſtum, and its Content in Wine Gallons.

OPERATION.

45)27.0(.6 *Quotient.*

Then againſt .60 (*Tab.* IV.), in the Column for the Fruſtums of *Parabolic Conoids*, we have .562, which being multiplied by the Difference of the Diameter (18), gives 10.116, to which add the leſs Diameter (27), and the Sum will be the required Mean-Diameter 37.116: — The Area in Wine Gallons, anſwering to this Diameter, is 4.683, which being multiplied by the Altitude . . 40

Gives 187.320 Wine Gallons, the Content ſought; *very nearly* the ſame as in the *Example*, *Pa.* 142.

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Altitude (or Length) 40 on C; then oppoſite 37.1, the above Mean-Diameter on D, we have 187.3 Wine Gallons, the Content on C.

Example 6.

Let the leſs Diameter of the Fruſtum of a Cone be 22, the greater Diameter 40, and the Altitude 60 Inches; required the Mean-Diameter and Content of the Fruſtum in Wine Gallons.

Operation.

40)22.00(.55 *Quotient.*

Then againſt .55 (*Tab.* IV. *Pa.* 191) in the Column for the Fruſtums of Cones, we have for a Multiplier .524; whence, by proceeding as in the laſt Example, the Mean-Diameter comes out 31.432, and therefore the Content is 201.48 Wine Gallons. (See *Pa.* 115.)

By the Sliding-Rule.

To the Wine Gauge-point on D, ſet the Altitude 60 on C; then againſt 31.43, the Mean-Diameter on D, we have 201.5 Gallons, the Content on C, *as before.*

The Mean-Diameters, found in the preceding Examples, may be alſo obtained by the 1ſt and 2d *Tables.* — Thus, againſt the laſt-mentioned Quotient .55 (in *Tab.* II.), we have for the Fruſtum of a Cone 7858

Multiplied by the greater Diam. 40

Gives the Mean-Diameter 31.4320, the ſame as above: Which, in this Caſe, is found with more Expedition than by the other Method; but it muſt be obſerved, that the ſaid Method is generally more conciſe than by the 1ſt and 2d Tables, by Reaſon that two Figures (and in ſome particular

Circumſtances *one)* taken for a Multiplier in the 3d and 4th Tables, will be as exact, as when four Figures are taken for a Multiplier in the 1ſt and 2d Tables.

Of the Conſtruction and Properties of the DIAGONAL ROD.

(See *Fig. Pa.* 160.)

The Diviſions graduated on this Rod, are founded upon the well-known Property of ſimilar Solids; namely, that their Contents are to one another as the Cubes of their homologous (or like) Sides, or Dimenſions.

If the Bung-Diameters, Head-Diameters, and Lengths of any two Caſks (of the ſame Variety) are in the ſame Proportion to each other, thoſe Caſks are then alike in Form, or ſimilar; and their Contents will be to each other, as the Cubes of their correſponding Dimenſions, and therefore (in this Caſe) as the Cubes of their Diagonals. Hence it is plain, that the original Conſtruction of the Diagonal Line was extremely eaſy: For the Bung and Head-Diameters, Length and Variety, of ſuch a Caſk as beſt agreed with the general Form of Caſks,* being firſt carefully taken in Inches and Tenths; then the Square of half the Length of the Caſk, being added to the Square of half

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* It is utterly impoſſible to inveſtigate what particular Form of a Caſk was firſt fixed upon, in the original Conſtruction of the Diagonal Line, even ſuppoſing the Property of the Curve of the Caſk known: It muſt be allowed that there is an infinite Number of different *Forms* and *Magnitudes* of each Variety of Caſks which have the very ſame Diagonal; nay, even in every cloſe Caſk but a cylindrical One, both the Diagonal and Content thereof may remain the ſame, and yet the Form of it, or the Ratio of the Bung and Head-Diameters, and Length, may vary; becauſe it is evident that the Diagonal and *one* other Dimenſion being known, are not ſufficient to *limit*, neither the *Figure*, nor *Magnitude* of the Caſk; whence it is plain, that, beſides

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beſides the Diagonal, there muſt be given another Dimenſion, in Order to obtain the Form of a Caſk of a *given* Magnitude; which, in the preſent Caſe, is to be a given Multiple of the Cube of the Diagonal:—Therefore if the Bung, or the Head, or the Ratio of the Bung and Head-Diameters is known; then with any of theſe, and a given Diagonal and Content anſwering thereto, the Form of the Caſk (ſuppoſing the Nature of the Curve known) will be eaſily determined; provided the greateſt Content that can be formed with the above *Data* is either equal to, or exceeds the Content anſwering to the propoſed Diagonal, or, which is the ſame Thing, to a given Multiple of its Cube; which Circumſtance is known to obtain in a ſpheroidical Caſk, in *every* Ratio of the Bung and Head-Diameters within a certain Limit; namely, when the Head-Diameter does not exceed Nine-tenths of the Bung-Diameter:—See my Queſtion in the *Ladies Diary* 1763; and its Solution in the ſubſequent *Diary*.

Now, in Order to determine which is the moſt general Form of ſpheroidical Caſks, to be met with in Practice, whoſe Contents will be truly obtained by the Diagonal Line; we muſt aſſume, to any given Content, either the Bung, or Head, or Ratio of the Bung and Head-Diameters (ſuch as is known to occur, according to the moſt general Form of Caſks); then the Queſtion becomes limited, and the Dimenſions of the Caſk may be found, ſo that the Diagonal Line will exhibit its true Content.

Let, for Inſtance, the Content of a ſpheroidical Caſk be $110\frac{1}{2}$ Wine Gallons (its Diagonal, on the Rod, will be 34.4 Inches, *nearly*), and let the Ratio of the Bung and Head-Diameters be as 1 to .85; moreover let the Bung-Diam. be denoted by x, and $p=.7854$: Then, by *Tab.* I. *Pa.* 182, we have for a Multiplier .9526, therefore $.9526x$ will expreſs the Mean-Diameter of the required Caſk; but $.925x = \frac{1}{2}$ the Sum of the Bung and Head-Diameters, conſequently the Length of the Caſk will be expreſſed by $2\sqrt{1183.36-\overline{.925x}|^{2}}$, whence $2\sqrt{1183.36-.855x^{2}}\times\overline{.9526x}|^{2}\times p=25525.5$, and $\therefore x=32$, the required Bung-Diameter; whence the Head-Diameter $(=32\times.85)=27.2$, and the Length $(=2\sqrt{1182.41-.855x^{2}})=35$, *very nearly*.

It is very evident that this Method is applicable, in the like Manner, to any of the other Varieties, as well as the ſpheroidical Form:—Suppoſe, for Inſtance, the Content of a Caſk of the 3d Variety to be 126 Wine Gallons, (whereof the Diagonal, on the Rod, is 35.9 Inches, *nearly*), and that the Ratio of the Bung and Head-Diameters be as 1 to .74. Then, by *Tab.* I. *Pa.* 182, againſt .74, we have for a Multiplier .8993; $\therefore 2\times\sqrt{\overline{35.9}|^{2}-\overline{\frac{1.74x}{2}}|^{2}}$ will expreſs the Length of the Caſk, and conſequently $2\sqrt{\overline{35.9}|^{2}-.757x^{2}}\times\overline{.8993x}|^{2}\times p$ $=126\times231=29106$, or $2\sqrt{1288.81-.757x^{2}}\times.8087x^{2}\times.7854=29106$, whence $x=31.3=$ the Bung-Diameter, and $\therefore 31.3\times.74=23.16=$ the Head-Diameter, conſequently the Length $(2\sqrt{1288.81-.757x^{2}})$ is 46.8.

OTHERWISE,

the Sum of the Bung and Head-Diameters, the Square Root of that Sum will give the *Meaſure* of the

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OTHERWISE, *without conſidering the Magnitude of the Caſk.*

If the Bung and Head-Diameters and Variety of a Caſk are known, we can readily determine the Length thereof, ſo that the Diagonal Line ſhall exhibit the Content of the Caſk.

Let the Bung and Head-Diameters be repreſented by *a* and *b* reſpectively, $r = .00272$ = the common Multiple of the Cube of the Diagonal, for Wine Gallons; alſo let A = the Area of a Circle in Wine Gallons, whoſe Diameter is = the Mean-Diameter of the Caſk; and $x = \frac{1}{2}$ the required Length thereof:

Then will $\sqrt{\overline{\frac{a+b}{2}}\Big|^2 + x^2}$ = the Diagonal; ∴ $\overline{\frac{a+b}{2}}\Big|^2 + x^2 \times \sqrt{\overline{\frac{a+b}{2}}\Big|^2 + x^2} \times r = A \times 2x.$

Now if, for Example, $a = 31.3$, $b = 23.16$, and ſuppoſing the Caſk to be of the 3d Variety; then, by *Tab.* I. *Pa.* 182, againſt .74 $\left(\frac{23.16}{31.3}\right)$, we have .8993 for a Multiplier, whence the Mean-Diameter is $(= .8993 \times 31.3) = 28.148$, ∴ $A = 2.69$ Wine Gallons; conſequently the above *general Equation* in Numbers, becomes $\overline{741.47 + x^2} \times \sqrt{741.47 + x^2} \times .00272 = 2.69 \times 2x$; whence $x = 23.4$, and ∴ the required Length $(2x)$ is 46.8, *the very ſame as before.*

COROLLARY.

It appears, from Sir *Iſaac Newton*'s Method of determining the Roots of Equations, that the laſt general Equation contains four *impoſſible Roots*, and the other two will be found to be real *affirmative Ones*: This Circumſtance holds good in every Ratio of the Bung and Head-Diameters, except when the ſaid Ratio approaches ſo near to that of Equality, as 1 to .898, in a ſpheroidical Caſk; or as 1 to .83 in a Caſk of the 3d Variety: — Whence it is plain, that, as the *Ratio* of the Bung and Head-Diameter approaches nearer and nearer to *theſe* abovementioned, the Limits of the ſaid *affirmative Roots* become narrower and narrower, 'till they (at laſt) coincide in the ſaid Ratios.

In the laſt Example, x has two affirmative Values; *i. e.* 23.4 and 16: Whence it appears, that if the Ratio of the Head, Bung, and Length, of a Caſk of the 3d Variety, be as 23.16, 31.3, and 46.8, or 23.16, 31.3, and 32 reſpectively, the Content thereof will be exhibited by the common Diagonal Line.

There are other general Methods for determining the Figure of a *Spheroidical Caſk*, whoſe Content will be obtained by the Diagonal Rod; The following Inveſtigation is on a Suppoſition, that the Content of the Caſk, and the Ratio of the Bung and Head-Diameters are known.

Let s = the Content of a Caſk in cubic Inches, d = its Diagonal (on the Rod), and let the given Ratio of Head and Bung-Diam. be as n to 1, $p = .7854$, and x =

the Diagonal Line; againſt *which* were placed the Contents of the Caſk, in Ale and Wine Gallons, found by the Rule agreeable to the Variety of the ſaid Caſk: Then it will hold, as the Cube of that Diagonal,

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the Semi-length: Then $\sqrt{d^2 - x^2} = \frac{1}{2}$ the Sum of the Bung and Head-Diameters; $\therefore \frac{2\sqrt{d^2-x^2}}{n+1} =$ the Bung-Diam, (for $n : 1 :: \text{Head} : \text{Bung}$, and by Compoſ. $n+1 : 1 :: H+B : B$), $\therefore \frac{2n\sqrt{d^2-x^2}}{n+1} =$ the Head; whence we have $\overline{\frac{8 \times \overline{d^2-x^2}}{\overline{n+1}|^2} + \frac{4n^2 \times \overline{d^2-x^2}}{\overline{n+1}|^2}} \times \frac{2px}{3} = a$: Now if a be expounded by $110\frac{1}{2}$ Wine Gallons, or 25525.5 cubic Inches, the correſponding Diagonal (d) by 34.4, and n by ,85; then the above general Equation in Numbers is $\frac{10.89 \times \overline{d^2-x^2}}{3.422} \times .5236x = 25525.5$, or $1183.36x - x^3 = 15376.8$, whence $x = 17.5$, *nearly*; and $\therefore \frac{2\sqrt{d^2-x^2}}{n+1} = 32$ the Bung, and $\frac{2n\sqrt{d^2-x^2}}{n+1} = 27.2$ the Head-Diameter, the very ſame as in *Pa.* 195.

Corollary.

Hence it appears, that, when n vaniſhes, the above Equation becomes $\overline{d^2-x^2} \times \frac{16px}{3} = a$, the Equation for a whole Spheroid; therefore, if $a = 29106$ (the cubic Inches in 126 Wine Gallons), and $d = 35.9$ the correſponding Diagonal, we ſhall have $x = 5.52$, or 32.82, for the Semi-lengths of a prolate and oblong Spheroid reſpectively, whence 35.47 and 14.55 $(\sqrt{d^2-x^2})$ are reſpectively the two Semi-diameters thereof; conſequently if the Ratio of the Axes of a prolate Spheroid be as 5.52 to 35.47, its Content will be *truly* exhibited by the Diagonal Line; but, to effect the ſame in an oblong Spheroid, the ſaid Ratio muſt be as 32.82 to 14.55.

N. B. In the above Equation for the ſpheroidical Caſk (as well as in *that* for a whole Spheroid), x has two poſitive Roots; and therefore the other Value of x (in the Equation $1183x - x^3 = 15376.8$) will come out 22.05, *nearly*; from whence the Bung $\left(\frac{2\sqrt{d^2-x^2}}{n+1}\right)$ and Head-Diameters $\left(\frac{2n\sqrt{d^2-x^2}}{n+1}\right)$ are found $= 28.5$ and 24.2 reſpectively.

Diagonal, is to the Content of the Caſk in Ale or Wine Gallons, ſo is the Cube of any other Number (or Diagonal) propoſed, to the Content of the Caſk in Ale and Wine Gallons, anſwering to that propoſed Diagonal: Whence it is evident, that the Cube of the Diagonal of *any* Caſk and its Content (according to this Conſtruction) are always in a conſtant Proportion; therefore, if the Content of the Caſk firſt found (or any Other) in Ale and Wine Gallons be divided by the Cube of its Diagonal, we ſhall obtain (.002228 and .00272) two general Multipliers, whereby the Cube of any propoſed Diagonal being multiplied, the Product will give the Content of the Caſk (on the Rod) in Ale and Wine Gallons reſpectively.

As the *Diagonal Line* is well known to be of general Uſe in practical Gauging; it may therefore not be amiſs to give a few eaſy Rules, whereby we ſhall be enabled to know when *it* may be applied with Certainty.

The Content of *every Spheroidical Caſk* will be obtained by the common Diagonal Line, if the Proportion of the Head and Bung-Diameters and Length be as 27.2, 32, and 49.5; or as 27.2, 32, and 35 reſpectively: Or, in other Words, if the Quotient of the Head-Diameter divided by the Bung-Diameter be .85, and the Quotient of the Head-Diameter divided by the Length be either .55, or .78, *nearly*.

But if the Quotient of the Head divided by the Bung-Diameter, of any *Spheroidical Caſk* whatever, be .85 (as above), and the Quotient of the Head-Diameter divided by the Length, ſhould be either leſs than .52, or greater than .78; then will the Diagonal exhibit more than the true Content of the Caſk; but if the ſaid Quotient is between .55 and

.78,

.78, the Diagonal will give leſs than the true Content.

Under the ſecond of the above-mentioned Forms *(or nearly)*, are comprehended all *Rum Puncheons*, *Herefordſhire*, *&c. Cyder Hogsheads*, and half *Hogsheads*; and many other Caſks to be met with in Practice.

The Diagonal Line will ſhew the true Content of every Caſk of the 3d *Variety*; whoſe Head, Bung-Diameter, and Length, are as 23.16, 31.3, and 46.8; or as 23.16, 31.3, and 32 reſpectively: Or, which is the ſame Thing, if the Quotient of the Head divided by the Bung-Diameter is .74, and the Quotient of the Head divided by the Length be either .5 or .73, *nearly*.

But if the Quotient of the Bung and Head-Diameters of any Caſk of the 3d Variety be .74, and the Quotient of the Head-Diameter divided by the Length be either leſs than .5, or greater than .73; then the Diagonal Line will exhibit more than the Content of the Caſk; but if the ſaid Quotient is between .5 and .73, then the Diagonal will ſhew leſs than the Content.

Hence it appears, that the true Content of *Liſbon* Wine Pipes (being of the 3d Variety) will be nearly obtained by the Diagonal Line, and alſo that it will exhibit more than the Content of *Port Pipes* (of the 3d Variety); becauſe the Quotient of the Head-Diameter divided by the Length is always leſs than .5.

It moreover appears, that a *Mountain Butt*, if it is of a ſpheroidical Form, will be ſomewhat undergauged by the Diagonal Line; but if *it* is of the 3d Variety, the Diagonal Line will then, *very nearly*, exhibit the true Content: For the Content of a Caſk of the 3d Variety, whoſe Head, Bung-Diameter, and Length, are as 26.6, 32, and 41.4

reſpectively (which is nearly the Form of *Mountain Butts)* will be obtained by the Diagonal Line.

Beſides the preceding Forms of Caſks, a vaſt Number of others might be pointed out, whoſe Contents would be truly exhibited by the Diagonal Line; but as the greateſt Part of thoſe Caſks are ſuch as never can occur in the Practice of Gauging, it may therefore ſuffice to put down the following Table, and to make one Remark farther, which will, I apprehend, be of Uſe to know: — That is, if the Quotient of the Head-Diameter divided by the Bung-Diameter be more than .9, in a *Spheroidical Caſk*, and greater than .83 in a Caſk of the *3d Variety*; then will the Diagonal Line ſhew more than the Content of the Caſk, let the Length thereof be what it will.

	Spheroidical Caſks:	*3d Variety.*
Quotient of the Head-Diam. divided by the Bung-Diameter.	Quotient of the Head-Diameter divided by the Length.	Quotient of the Head-Diameter divided by the Length.
.65	.36 *or* .9	.416 *or* .75, *nearly.*
.7	.4 *or* .87	.47 *or* .74
.74	.436 *or* .85	.5 *or* .73
.8	.49 *or* .8	.57 *or* .67
.83		.64 ———
.85	.55 *or* .78, *nearly.*	
.9. *nearly.*	.67 ———	

This little Table ſhews, at one View, eleven different Forms of *Spheroidical Caſks*, and nine different Forms of the *3d Variety*, whoſe Contents will be truly obtained by the common Diagonal Line: And, from what is delivered in the foregoing Pages, it will be eaſy to know whether the Diagonal Line exhibits more or leſs than the true Content of the Caſk; provided the *Quotient* of the Head-Diameter thereof divided by the Bung-Diameter, is equal to any of *theſe* in the preceding Table.

SECTION XI.

OF ULLAGING OF CASKS.

THE Method now practiſed in the Ullaging of Caſks, whether lying or ſtanding, is by the Lines of Segments on the Sliding-Rule (deſcribed in *Pa.* 32, *&c.*) Though other Methods may indeed be given, far more general and accurate; yet there are none, that have occurred to me, but what will, perhaps, be thought too tedious for practical Uſe.

The ingenious Mr. *William Yeo* (in a whole Treatiſe upon Ullaging, publiſhed in the Year 1749) has computed very accurate, extenſive, and familiar Tables of Segments, not only for one particular Sort, but for eight different Forms, both of ſtanding and lying ſpheroidical Caſks: From theſe *Tables* we can readily determine what two Forms of ſpheroidical Caſks agree, *very nearly*, with the Lines of Segments on the Sliding-Rule: —*Viz.* The Ullage of every ſtanding ſpheroidical Caſk, whereof the Quotient of the Head divided by the Bung-Diameter is .82,[*] will be, *very nearly*,

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[*] It appears, in the Tables above cited, that the 6th Column for ſtanding ſpheroidical Caſks (where the Head divided by the Bung-Diameter is .81, .82, .83, or .84) anſwers beſt to the Line S.S on the Sliding-Rule; therefore let .82 (which is near the Mean of the Four) be taken for the Quotient of the Head divided by the Bung-Diameter; then will the Ullage of *any* ſtanding *Spheroidical Caſk*, having that Property, be (*very nearly*) obtained by the Sliding-Rule, *let the Length of the Caſk be what it will*:—For we ſhall prove in a *Corollary* farther on (*Pa.* 207.) that in *any* two ſtanding *Spheroidical Caſks*, having the ſame Ratio of Bung and Head-Diameters, and alſo the Quotient of

obtained by the Line S.S, and thoſe marked A and B on the Sliding-Rule: And the Ullage of a lying ſpheroidical Caſk will be *nearly* found by the Sliding-Rule, when the Quotient of the Head divided by the Bung Diameter is .75, and the Quotient of the Head-Diameter divided by the Length is .5.† Moreover, it is eaſy to perceive, from the forementioned Tables, that, in any ſtanding ſpheroidical Caſk, if the Quotient of the Head divided by the Bung-Diameter is leſs than .82, the Ullage then obtained by the Lines S.S and A and B, will be *too much*, if the Caſk is leſs than half full; and *too little* if above half full: But, on the contrary, if the ſaid Quotient is greater than .82; then the Ullage, found as above, will be *too little*, if the Caſk is leſs than half full; and *too much*, if more than half full.

Again,

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of the wet Inches of each Caſk, divided by its reſpective Length, equal to each other, the Ullages of thoſe two Caſks will be in the Ratio of their whole Contents.

But, with Regard to lying Caſks, the Caſe is different; for the Ullages of any two lying *Spheroidical Caſks*, having different Lengths, and the Bung and Head-Diameters and alſo the wet Inches the ſame, will not (except when the Caſks are *exactly* half full) be to each other in the Ratio of the whole Contents of thoſe Caſks: Hence it appears, that the Line S.L on the Sliding-Rule can only anſwer to *one* particular Form of *Spheroidical Caſks*, i. e. to ſuch, whoſe Head, Bung, and Length, are in ſome conſtant Ratio: In Order to determine which, proceed as follows.

† In the foreſaid Tables, for lying ſpheroidical Caſks, it appears that the Segments which anſwer neareſt to thoſe on the Sliding-Rule, ſtand under theſe Ratios of the Bung and Head-Diameters, *viz.* .74, .75, and .76: Suppoſe, for Inſtance, we take .75 (as being the Mean); then, by the well-known *Theorem* for ſpheroidical Caſks, $\overline{2+.75^2} \times .7854 \times \frac{l}{3} = 1$, ∴ $.6708\,l = 1$, and $l = \frac{1}{.67} = 1.5$, the required Length, *nearly*.

Hence, the Quotient of the Head divided by the Bung-Diameter is .75, and the Quotient of the Head-Diameter divided by the Length $\left(\frac{.75}{1.5}\right)$ is .5.

Or, which is the ſame Thing, the Head, Bung-Diameter and Length, are as 24, 32, and 48; whence the Ullage of every lying *Spheroidical Caſk*, having this Property, will be *nearly* obtained by the Sliding-Rule.

Again, it appears by theſe Tables, that in a lying ſpheroidical Caſk, if the Quotient of the Head divided by the Bung-Diameter be leſs than .75, and the Quotient of the Head-Diameter divided by the Length be leſs than .5, the Ullage found by the Line S.L and the Lines A and B, will be *too much*, if the Caſk is leſs than half full; and *too little*, if above half full; But if the ſaid Quotients are greater than .75 and .5; then the Ullage, found as above, will be *too little*, if the Caſk is leſs than half full; and *too much*, if above half full.

It may be proper to obſerve to the practical Reader, that, in ullaging by the Sliding-Rule, we are much more ſubject to Error in lying, than in ſtanding Caſks.

To find the Ullage of a Caſk by the Sliding-Rule.

Prop. I.

The Length (or Bung-Diameter), wet Inches, and Content, of a ſtanding (or lying) Caſk being given; to find the Ullage thereof.

General Rule.

To 100 on S.S (or S.L) ſet the Length (or Bung-Diameter) on the Slide N; then againſt the wet Inches on N, is the *Segment* on S.S, if a ſtanding Caſk; or on S.L, if a lying Caſk: Again, to 100 on A, ſet the whole Content on B; then oppoſite the ſaid Segment on A, is the required Ullage on B.

EXAMPLE 1.

Let the Content of a ſtanding Caſk be 142 Gallons, the Length 52, and the wet Inches 20; to find the Ullage, or Quantity of Liquor in the Caſk.

To 100 on S.S, ſet the Length 52 on N; then againſt the wet Inches 20 on N, is 37 the Segment on S.S: Again, to 100 on A, ſet the whole Content 142 on B; then againſt the above Segment 37 on A, is 52¼ Gallons, the required Ullage on B.

EXAMPLE 2.

Suppoſe the Content of a lying Caſk to be 112 Gallons, the Bung-Diameter 33, and the wet Inches 22.5; to find the Ullage.

To 100 on S.L, ſet the Bung-Diameter 33 on N; then againſt the wet Inches 22.5 on N, is 74.5 the Segment on S.L: — Again, to 100 on A, ſet the Content of the Caſk 112 on B; then againſt the ſaid Segment 74.5 on A, is 83.5 Gallons, *very nearly*, on B.

After the Segment of a Caſk (either ſtanding or lying) is found, the Reſult, from the Remainder of the Operation, will be the very ſame as above; if to 100 on A, we ſet the Segment (inſtead of the Content of the Caſk) on B, and look againſt the ſaid Content on A, for the Ullage on B.[*]

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[*] The Ullages of two ſtanding ſpheroidical Caſks (whoſe Bung and Head-Diameters are to each other in the ſame Ratio, and the wet Inches of each Caſk in the Ratio of their Lengths) will be to each other as their whole Contents (ſee

Pa.

In the laſt Example, the Segment was found to be 74.5: Therefore to 100 on A, ſet 74.5 on B; then againſt the Content 112 on A, is 83.5, *very nearly*, on B, *the ſame as before.*

It may not be amiſs to obſerve, that after the Segment of any Caſk is found (as above) by the Lines S.S and S.L, the Reſt of the Operation may be performed, without the Lines A and B, by only multiplying the whole Content of the Caſk by the ſaid Segment; the Product thereof, after pointing off two Decimals more than are contained in both the Content and Segment, will be the required Ullage.

In the preceding Example, the Content of the Caſk is 112 Gallons, which being multiplied by the Segment 74.5, the Product is 8344.0; therefore, by cutting off two Decimals more, we have 83.440 Gallons, the required Ullage, *the ſame as above.*

If it was required to find the Vacuity, or the Quantity of Liquor drawn out of a Caſk, the Method of Operation will be the very ſame as any of thoſe given above; only obſerve to uſe the dry Inches, inſtead of the wet.

Pa. 207): Therefore let a denote the Content of a Caſk, whoſe Ullage is ſought, and b the Segment (or correſponding Ullage) of a Caſk, whoſe Content is 100 Gallons; we ſhall then have, $100 : a :: b : \frac{a \times b}{100}$, or, alternately, $100 : b :: a : \frac{a \times b}{100}$: Whence it is plain, that the Reſult will be the very ſame, whether a or b (on B) is ſet to 100 on A: Moreover, dividing the Product $(a \times b)$ by 100, is manifeſtly the ſame, as cutting off two Decimals more than are contained in the Factors a and b.

The ſame is to be obſerved in two lying Caſks, only thoſe indeed muſt be ſimilar in every Reſpect, and conſequently the Segments (or Ullages) will be ſo too, provided the wet Inches and Bung-Diameter of each Caſk are to each other in the ſame Ratio.

PROP. II.

The Bung and Head-Diameters, Length, and wet Inches of any ſtanding Spheroidical Caſk *being given*; *to determine the Ullage thereof.*

RULE.

1. Divide the Square of the wet Inches by three times the Square of half the Length of the Caſk, to the Quotient add Unity, and from this Sum ſubtract the Quotient of the wet Inches divided by half the Length of the Caſk, and *note* the Difference.

2. Multiply the Sum of the two given Diameters by their Difference, that Product multiply by the wet Inches, and this Product multiply by the above *noted* Difference; then let this laſt Product be ſubtracted from the Square of the Bung-Diameter multiplied by the wet Inches,† and the Remainder being multiplied by $\left\{\begin{matrix}.0027851 \text{ for Ale Gallons}\\ .0034 \text{ for Wine Gallons}\end{matrix}\right\}$, or divided by $\left\{\begin{matrix}359 \text{ for Ale Gallons}\\ 294 \text{ for Wine Gallons}\end{matrix}\right\}$ the Product, or Quotient, gives the required Ullage.

EXAMPLE.

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† Let the Bung-Diameter (CD, *Fig.* XIV), Head-Diameter (EF or GH), and half the Length (OL or OM) of a *Spheroidical Caſk* (in *Inches*), be denoted by b, b and l reſpectively; alſo let $p = .78539$, the Semi-tranſverſe Axis (AO) of the whole Spheroid $= v$, and the variable Diſtance $Lm = x$: Then, by the Property of the Curve, we have $\frac{pb^2}{v^2} \times \overline{v-l+x} \times \overline{v+l-x}$ $\left(= \frac{pb^2}{v^2} \times \overline{v^2-l^2+2lx-x^2}\right) =$ the Meaſure of the Section mr; therefore $\frac{pb^2}{v^2} \times \overline{v^2\dot{x}-l^2\dot{x}+2lx\dot{x}-x^2\dot{x}}$ is the Fluxion of the

EXAMPLE.

Let the Bung-Diameter of a ſpheroidical Caſk be 35 Inches, the Head-Diameter 28.7, the Length 40, and the wet Inches 30; required the Ullage, in Ale and Wine Gallons.

OPERATION.

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the required Solid, whoſe Fluent is $\frac{pb^2}{v^2} \times \overline{v^2x - l^2x + lx^2 - \frac{x^3}{3}}$; but, by the Nature of the Curve, $b^2 = \frac{b^2}{v^2} \times \overline{v^2 - l^2}$, $\therefore v^2 = \frac{b^2l^2}{b^2 - b^2}$, which being ſubſtituted (for v^2) in the above Expreſſion, we get $\frac{pb^2}{\frac{b^2l^2}{b^2-b^2}} \times \overline{\frac{b^2l^2x}{b^2-b^2} - l^2x + lx^2 - \frac{x^3}{3}}$ $\left(= \frac{p \times \overline{b^2 - b^2}}{l^2} \times \overline{\frac{b^2l^2x}{b^2-b^2} - l^2x + lx^2 - \frac{x^3}{3}} = pb^2x + \frac{p \times \overline{b^2 - b^2}}{l^2} \times \overline{lx^2 - l^2x - \frac{x^3}{3}}\right) = pb^2x + px \times \overline{b^2 - b^2} \times \overline{\frac{x}{l} - 1 - \frac{x^2}{3l^2}}$, the Content (in Inches) of the variable Part FE*mr*. Q. E. I.

COROLLARY I.

When, in the foregoing Expreſſion for the Ullage, $x = 2l$; we then get $2pb^2l + 2pl \times \frac{b^2 - b^2}{3}$, or its Equal, $\frac{2pl}{3} \times \overline{2b^2 + b^2}$, for the whole Content of a ſpheroidical Caſk, in *cubic Inches*.

COROLLARY 2.

Hence we can eaſily deduce the Reaſon of that Aſſertion in the *Note*, *Pa.* 201: — For it is evident (ſuppoſing $\frac{x}{l}$ to remain the ſame) that $\frac{x}{l} - 1 - \frac{x^2}{3l^2}$ will be a conſtant Quantity; and therefore ($\dot{b}$ and b being conſtant) the Expreſſion $(pb^2x + px \times \overline{b^2 - b^2} \times \overline{\frac{x}{l} - 1 - \frac{x^2}{3l^2}})$ for the Ullage, let l be what it will, is evidently as x, the Altitude of the Segment; but x and l (by Hypotheſis) are in a conſtant Ratio, and $\therefore$ the above Expreſſion (in ſuch Caſe)

OPERATION.

The Square of 30 (the wet Inches) is 900, which being divided by 3 times the Square of 20, half the Length of the Caſk (*viz.* 1200), the Quotient will be .75, to which add Unity, and it becomes 1.75
Subt. the Quotient of 30 divided by 20, *viz.* 1.5

Difference *note* .25

The

Caſe) will be as l, the whole Length of the Caſk, which is manifeſtly as the whole Content thereof, b and b remaining the ſame; conſequently the above Expreſſion (for the Ullage of any ſtanding *Spheroidical Cask*) when b, b and $\frac{x}{l}$ remain the ſame, will be to the whole Content thereof in a conſtant Ratio, let its Length be what it will.

OTHERWISE, *more generally.*

Let b and b denote any two Numbers whatever, in a conſtant Ratio to each other, and let $\frac{x}{l}$ be ſuppoſed a conſtant Quantity: Put $\frac{x}{l} - 1 - \frac{x^2}{3l^2}$ (being in this Caſe conſtant) $= n$; then the foregoing Expreſſion for the Ullage becomes $px \times \overline{b^2 + \overline{\overline{b^2 - b^2}} \times n}$; but the whole Content of the Caſk is (by *Cor.* 1.) expreſſed by $\frac{2pl}{3} \times \overline{2b^2 + b^2}$, we muſt therefore prove whether or not $px \times \overline{b^2 + \overline{\overline{b^2 - b^2}} \times n}$ is to $\frac{2pl}{3} \times \overline{2b^2 + b^2}$ in a conſtant Ratio, under the above-mentioned Circumſtances: Firſt then (by Hypotheſis) $px : \frac{2pl}{3}$ (or $x : \frac{2l}{3}$) in a conſtant Ratio; it therefore now remains to prove that $b^2 + \overline{b^2 - b^2} \times n$ (or $b \times b + \overline{b+b} \times \overline{b-b} \times n$) is to $2b \times b + b \times b$ in a conſtant Ratio: Now it is evident, that every Factor (*i. e.* b, $b+b$, $b-b$, and b) contained in the Terms of the Ratio will be equally affected by any Multiple, or Part, of b (and b) being taken; ∴ the four Quantities, or Rectangles ($b \times b$, $\overline{\overline{b+b} \times \overline{b-b}} \times n$, $2b \times b$, and $b \times b$), *viz.* two in each Term of the propoſed Ratio, will be each of them augmented,

The Sum of the two Diameters is 63.7
Multiplied by their Difference 6.3

1911
3822

Gives 401.31
Multiplied by the wet Inches 30

Gives 12039.30
Multiplied by the above *noted* Diff. .25

The Product is 3009.825

The Square of the Bung-Diam. (35) is 1225
Multiplied by the wet Inches 30

Product is 36750
From which take the laſt Product 3009.825

Remains 33740.175, which being multiplied (or divided) according to the preceding Rule, we ſhall have 94 Ale Gallons

E e and

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augmented, or diminiſhed, alike, and conſequently $b\times b+\overline{b+b}\times\overline{b-b}\times n : 2b\times b+b\times b$ in a conſtant Ratio; this being multiplied by two Factors, which are (by Hypotheſis) alſo in a conſtant Ratio to each other (*i. e.* px and $\frac{2pl}{x}$), the Products muſt alſo be in a conſtant Ratio; that is, $px\times\overline{b^2+\overline{b^2-b^2}\times n} : \frac{2pl}{3}\times\overline{2b^2+b^2}$ in a conſtant Ratio, let b and b be what they will (in a conſtant Ratio), and $\frac{x}{l}$ a conſtant Quantity:—That is, in Words, let any two ſtanding ſpheroidical Caſks be taken, whoſe Bung-Diameters and Head-Diameters are in the ſame Ratio to each other, and alſo let the wet Inches of each Caſk be to each other in the Ratio of their Lengths; then will the Contents of thoſe Ullages be to each other in the Ratio of the whole Contents of the Caſks, let their Lengths be what they will.

and 114.7 Wine Gallons, the Contents of the required Ullage.

The whole Content of the foregoing Caſk (by the *Rule* in *Pa.* 161) is 148.5 Wine, and 121.5 Ale Gallons; whence, by the *Sliding-Rule*, the Contents of the Ullage will come out the very ſame as thoſe above: The Reaſon whereof is, becauſe if the Head-Diameter (28.7) is divided by the Bung-Diameter (35), the Quotient will be .82; ſee *Pa.* 201.

The preceding Rule is *ſtrictly true* for determining the Ullage of any ſtanding ſpheroidical Caſk whatever; and, though rather too tedious for ordinary Practice, will, I apprehend, be found more expeditious than any *General Rule* hitherto given for that Purpoſe; there being no Neceſſity, by this Method, for previouſly finding the Content of the Caſk, before *that* of the Ullage.

But if there be known (beſides the Dimenſions given in the foregoing Propoſition) the Diameter of the Liquor's Surface, we can readily determine the Ullage of any upright Caſk, let its Variety be what it will.

For let a Mean-Diameter, and conſequently the Area in Ale and Wine Gallons, corresponding to the Bung-Diameter and the Diameter of the Liquor's Surface, be found, agreeable to the Variety of the Caſk, as already taught in *Sect.* X; then this Area being multiplied by the Diſtance of the Surface of the Liquor from the Bung-Diameter, and that Product added to, or ſubtracted from half the Content of the Caſk, according as it is *more* or *leſs* than half full; the Sum, or Difference, will be the required Ullage.

From what has been delivered (*Sect.* IX. *Pa.* 171) we might eaſily deduce Rules for computing the Diameter of the Surface of the Liquor, at any given Altitude of an upright ſpheroidical Caſk, or that of a parabolic Spindle: But the following Method is far more expeditious, and will be ſufficiently exact, for any of the three Varieties.

Suppoſe, for Example, the Diſtance *br* (ſee the *Fig. Pa.* 157) to repreſent the wet Inches; then carefully meaſure the perpendicular Diſtance *ad*, the Double whereof being taken from the Bung-Diameter AB, leaves the Diameter of the Liquor's Surface.

Let, for Inſtance, the Bung and Head-Diameters, Length and wet Inches of a ſpheroidical Caſk be the ſame as in the preceding Example, alſo let the Diameter of the Liquor's Surface be 33.5 Inches, found (in this Caſe) by the Rule in *Pa.* 171; required the Ullage in Wine Gallons.

OPERATION.

Bung. Head. Quot.
35) 33.50 (.96, *nearly*.

Then (by the Method in *Pa.* 186) againſt .96, in the Column for ſpheroidical Caſks, we have 67
Which being multiplied by the Difference of the Diameters . . . . } 1.5

335
67

Gives 1.005
Add the Head-Diameter 33.5

Mean-Diameter 34.505, the

Area whereof in Wine Gallons is 4.04, *&c.*
Multiply by the Diſtance of the Liquor's Surface from the B. Diam. } 10

Gives 40.40
Add half the Content (ſee *Pa.* 210) 74.25

Gives the Ullage 114.65 Wine Gallons, *the ſame as before.*

The Buſineſs of finding a Mean-Diameter (*Sect.* X.) being now rendered very exact, expeditious, and general, for any of the three Varieties; it is therefore preſumed, that the preceding Method of computing, by the Pen, the Ullage of any ſtanding Caſk (and alſo that which follows for lying Caſks), will be found preferable to any *other* that can be given.

PROP. III.

The Bung and Head-Diameters, Length, Variety, and wet Inches of any lying Caſk (leſs than half full) being known; to find the Quantity of Liquor therein, in Ale and Wine Gallons.

RULE.

Let the Mean-Diameter be found (ſee *Sect.* X.) agreeable to the propoſed Variety of the Caſk: From the wet Inches ſubtract half the Difference between the Bung and Mean-Diameter, and divide the Remainder (with Cyphers annexed, ſee the Rule in *Pa.* 93) by the Mean-Diameter; then, againſt the Quotient, under the Letter V.S, in the Table of the Areas of the Segments of a Circle, we have a Decimal Fraction, which being multiplied by the Square of the Mean-Diameter, that Product multiply by the Length of the Caſk, and this laſt Product divide by 282 for Ale, and 231 for Wine Gallons, the Quotient will give the Ullage ſought.*

Example.

Suppoſe the Bung-Diameter of a ſpheroidical Caſk is 32 Inches, the Head-Diameter 24, the Length 48, and the wet Inches 14; required the Ullage in Ale and Wine Gallons.

Operation.

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* Let BCEF (*Fig.* XV.) repreſent a lying Caſk, ab its Mean-Diameter, Ad the wet Inches: — Then, ſuppoſing RA drawn parallel to the Axis sn, it is plain that Ae (= bc) = $\frac{AD - ab}{2}$ = ½ the Difference between the Bung and Mean-Diameter; ∴ Ad (cr) — Ae = br = the verſed Sine of the Segment of a Circle whoſe Diameter is ab. Now let the Meaſure of a Segment of a Circle, whoſe Diameter is Unity, (in the preſent Caſe = 1 Inch) be denoted by A, the Mean-Diameter $ab = b$, and the Length sn (= vw) = l; then, by the *Theorem*, *Pa.* 91, $1^2 : b^2 :: A : b^2 \times A$ = the Meaſure of a Segment of a Circle (whoſe Diameter is ab) ſimilar to that of A; conſequently $b^2 A \times l$ = the Meaſure (in Inches) of the Ullage AB$vw$F. Q. E. I.

Corollary.

If A repreſents the Meaſure of a Segment of a Circle, whoſe *Area* is Unity (*i. e.* one Inch), b and l as before: — Then, becauſe the Areas of Circles are as the Squares of their Diameters, we have $1 : b^2 \times .7854 :: A : .7854 b^2 \times A$ = the Meaſure of a Segment, ſimilar to that of A, ∴ $.7854 b^2 l \times A$ = the Meaſure of the Ullage AB$vw$F in Inches, *nearly*; that is, the Segment in the Table, in *Everard*'s Gauging (where the Area of the Circle is Unity), being multiplied by the whole Content of the Caſk, gives the required Ullage: But the Methods exhibited above are more expeditious; becauſe we are, by thoſe Methods, under no Neceſſity of, previouſly, finding the whole Content of the Caſk; and moreover, the Ullage may be obtained with the ſame Expedition, whether the Caſk is more or leſs than half full, provided the Table of Segments of a Circle was continued to 1000 Places, or to the Area of the whole Circle, *i. e.* to .785398, &c. Which indeed may be very eaſily effected, in the following Manner.

From .785398 (the Area of a Circle whoſe Diameter is Unity) ſubtract, ſucceſſively, the Segments anſwering to the verſed Sines .499, .498, .497, and .496, &c. and the Remainders will ſhew, reſpectively, the Meaſures of the Segments correſponding to .501, .502, .503, and .504, &c. Parts of Unity, or the Diameter of the Circle.

OPERATION.

The Bung-Diameter 32
TheM.Diam.found by theRule, *P*.186 29.57

Difference 2.43

Half Difference is 1.21, which being taken from 14, the wet Inches, leaves 12.79; then

M. Diam. M. Wet. Quot.
29.57)12.79000(.432

Againſt .432, under the Letters V.S, in the Table of Segments of a Circle, is . .324909
Multiplied by the Square of 29.57, the Mean-Diameter, *viz.* . . } 874.38

Product is 284.0939
Multiplied by the Length 48

22727512
11363756

Product 13636.5072

282)13636.507(48.356 Ale Gallons.
231)13636.507(59.032 Wine Gallons.

But if it be required to find the Quantity of Liquor drawn out of any lying Caſk, when leſs than half full, or remaining in it, when above half full, proceed as follows.

Find, by the preceding Rule, the circular Segment in the *Table* correſponding to the wet Part of the Caſk, when leſs than half full, or to the dry Part, when more than Half; which being ſubtracted from .785398, the Remainder will be the Meaſure of a Segment ſimilar to the wet and dry Part of the Caſk reſpectively; this being multiplied and divided, as in the preceding Rule, gives the Ullage ſought.

Let it be required to find, in the foregoing Example, the Vacuity, or Quantity of Liquor drawn out of the Caſk; the Operation will be as follows.

From the Area of a Circle, whoſe Diameter is Unity, *viz.*	.785398
Subtract the Segment anſwering to the wet Part (ſee *Pa.* 214.) . . .	.324909
Leaves a Segment ſimilar to that correſponding to the dry Part of the Caſk	.460489
Multiplied by the Square of the Mean-Diameter, *viz.*	874.38
Product is	402.6423
Multiplied by the Length	48
	32211384
	16105692
Product	19326.8304

282)19326.830(68.534 Ale Gallons.
231(19326.830(83.665 Wine Gallons.

The Method of Operation for finding the Quantity of Liquor in a lying Caſk, more than half full, is the very ſame as *that* given above; except, that the Segment (in the Table) muſt be found for the dry, inſtead of the wet Inches.

The foregoing *Method* of ullaging a lying Caſk, though not ſtrictly true, is more exact than any *other* that has yet occurred to me, and may be applied, with equal Facility, to any of the three Varieties; becauſe it chiefly depends on the Mean-Diameter, which is now obtained with great Eaſe and Exactneſs, let the Variety of the Caſk be what it will: (See *Sect.* X.)—It may, however, be proper to obſerve here, that the Quantity of Liquor in a lying Caſk, obtained by the preceding Methods, will be *too much*, if it be more than half full; and *too little*, if leſs than half full; but the greateſt Error that can poſſibly happen, either in Exceſs or Defect, will be wholly inconſiderable in the Practice of Gauging.[*]

Note. It may be proper to mention a Circumſtance, which was accidentally omitted in *Sect.* X. *Viz.* If the exact *Quotient* of the Head (or leſs) Diameter divided by the Bung (or greater) Diameter cannot be found in the *Tables* (ſee *Pa.* 185, *&c.*); then the Mean-Diameter will differ a *ſmall* Matter from the Truth; but the greateſt Difference that can ever happen, by the *Tables*, will be wholly inconſiderable in Practice, and that Difference will even become leſs, if we obſerve to take out the Multiplier which anſwers the neareſt to the ſaid *Quotient*: Thus, let the Head-Diameter be 21.7 and the Bung 32 Inches; theſe being divided as above, the *Quotient* will be .678, *&c.*; therefore the *Number* againſt .68 (in any of the *Tables*) will be more exact for a *Multiplier*, than *that* againſt .67.

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[*] It is very evident (ſee *Fig.* XV.) that, when the wet Inches are equal to (or leſs than *Ae*) half the Difference between the Bung and Mean-Diameter, the verſed Sine, and, conſequently, the mean wet Inches vaniſh; and therefore the Quantity of Liquor in the Caſk (according to this Method of finding the Ullage) will be = 0, when the wet Inches are equal to *Ae*, or leſs than that Diſtance: Which is *abſurd.*—Whence it follows, that the Quantity of Liquor in a Caſk (obtained by this Method) will be a *ſmall Matter too little*, if leſs than half full; and *too much*, when above half full.

SECTION XII.

Of *meaſuring* Curve-lined Planes, *by Approximation.*

THE general Method of approximating the Areas of curvilineal Planes, by Means of any given Number of equidiſtant perpendicular Ordinates (or Diameters), was firſt demonſtrated by the moſt illuſtrious Newton, and is well known to be a Subject of very great Importance in ſpeculative Mathematics.

And although this general Method has already been adapted to the preſent Subject (particularly, firſt of all, by that excellent Mathematician Mr. *Robert Shirtcliffe*, in his Theory and Practice of Gauging, and afterwards by my late worthy and ingenious Friend, Mr. *Samuel Farrer*, in the Appendix to *Overley*'s Gauging), yet we find it has not ſufficiently merited the Attention of every practical Gauger, which, it is preſumed, is owing to the Tediousneſs of the Rules hitherto laid down.

For this Reaſon, I have endeavoured to put the Matter in as *clear* and *practical* a Light, as the Nature of the Subject can poſſibly admit of, and have illuſtrated the ſame with ſuitable Examples: —— And moreover, to oblige the ſpeculative Readers, I have given, in the ſubjoined Note* the

F f Demonſtration

Proposition.

* *Suppoſe the* black Curve-line vnp (ſee Fig. XVI.) *to repreſent a ſmall Portion of any Curve whatever, and the* dotted Line vnp *a ſmall Portion of a*

common

Demonſtration of this Method; (which indeed does not eſſentially differ, except in one Circumſtance, from

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common parabolic Curve; each paſſing through the Extremities of the three equidiſtant Perpendiculars Av, Bn, Cp: *To find an Expreſſion in Terms of thoſe three Ordinates* (or Diameters), *and their common Diſtance* AB (BC, &c.), *that ſhall* accurately *meaſure the parabolic Space, and conſequently that comprehended by the black Curve-line* vnp, *the Right-line* AC, *and the Perpendiculars* Av, Cp, indefinitely near.

Suppoſe the Axis (PQ) of the parabolic Curve, to be parallel to the Ordinates of the propoſed Curve, draw the Right-line (or Ordinate) vwp, and parallel thereto draw MnS, which is well known to be a Tangent to the parabolic Curve, at the Point n; produce Av and Cp to meet MS in m and s: Then becauſe it is proved, by the Writers on Fluxions, that a Parabola is two-thirds of a Rectangle of the ſame Baſe and Altitude; it follows, from the very ſame Principles, that the parabolic Area $vnpwv$, is two-thirds of the Parallelogram $vmsp$. — Now it is evident, from common Geometry, that Bn $\left(= \frac{Am + Cs}{2}\right) \times 2AB$ is equal to the Area of the quadrilateral Space $AmsC$, and alſo that the Area of the Quadrilateral $AvwpC$ is expreſſed by $\overline{Av + Cp}$ $(2Bw)$ $\times AB$; moreover it is plain, that the Quadrilateral $AmsC$ is greater than the parabolic Area $AvnpC$, by *exactly* half what the Quadrilateral $AvwpC$ wants of that Area; conſequently twice $AmsC$ (= twice the parabolic Space $AvnpC + vnpw$, or twice $vmsp$) added to $AvwpC$, gives three times the parabolic Area $AvnpC$; which Area alone, will thereſore be, *accurately*, expreſſed by $\frac{2Bn \times 2AB + \overline{Av + Cp} \times AB}{3}$, or $\overline{Av + 4Bn + Cp} \times \frac{AB}{3}$; and thereſore, when the three Ordinates (or Perpendiculars) are taken pretty near to each other, the common parabolic Curve paſſing through their Extremities, will, *very nearly*, coincide with any other Curve, paſſing through the ſame three Points: Becauſe, as a *Parabolic Curve* has an infinite Variation of Curvature, it may be juſtly conceived to be, *very nearly*, coincident with any *other Curve* for a ſmall Diſtance: Whence it is plain, the above Expreſſion $\left(\overline{Av + 4Bn + Cp} \times \frac{AB}{3}\right)$ will exhibit the *accurate Meaſure* of the parabolic Space $AvnpC$; and conſequently *(very nearly)* of *that* bounded by the Right-line AC, the Perpendiculars Av, Cp; and any Curve-line paſſing through the Extremities *(v, n, and p)* of the three equidiſtant Perpendiculars Av, Bn, and Cp. Q. E. D.

COROLLARY I.

From hence it is eaſy to deduce a general Rule for determining, *very nearly*, with any odd Number of equidiſtant Ordinates, or Perpendiculars whatever, the Meaſure of any curvilineal Plane, bounded at its Ends by Right-lines, parallel from what is given by that incomparable and moſt profound Mathematician, the late Mr. THOMAS SIMPSON, in his *Diſſertations*, *Pa.* 109.)

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parallel to each other: For let the perpendicular Diſtance of thoſe two given parallel Lines Av, Gb, be divided into equal Parts, by any odd Number of Perpendiculars, Bn, Cp, De, &c. which in ſome Curves are conſidered as Ordinates, and in *others* (particularly the *Parabola*) as Diameters (and the greater the Number, the greater is the Degree of Accuracy): Then by the very ſame Reaſoning, as we found $\overline{Av+4Bn+Cp}\times\frac{AB}{3}$, for the Area of $AvnpC$, we get $\overline{Cp+4De+Ef}\times\frac{CD}{3}\left(\frac{AB}{3}\right)$ for the Area of $CpefE$, and likewiſe that of $EfgbG=\overline{Ef+4Fg+Gb}\times\frac{AB}{3}$, &c; therefore the Sum of thoſe Areas (each having one common Multiplier) will be expreſſed by $\frac{AB}{3}\times\overline{Av+4Bn+2Cp+4De+2Ef+4Fg+Gb}=$ the Area of the curvilineal Plane $AvnpefgbG$, *nearly*: Whence the general Rule is manifeſt.

COROLLARY 2.

If, at the Extremity of the curvilineal Space whoſe Area is ſought, the Ordinate be ſuppoſed to vaniſh; then (Av being Nothing) we have $\overline{4Bn+2Cp+4De+2Ef+4Fg+Gb+\&c.}\times\frac{AB}{3}$ a general Expreſſion for approximating, with any even Number of equidiſtant Perpendiculars, the Area of any curvilineal Plane, bounded by two perpendicular Right-lines and a Curve. The above Expreſſion, in Words, will be as follows:—*To four times the Sum of the 1ſt, 3d, and 5th,* &c. *Perpendiculars* (beginning at the leaſt) *add the laſt, and alſo Double the Sum of all the Reſt; this Total being multiplied by ⅓d of their common Diſtance, the Product will be the required Area,* nearly.

COROLLARY 3.

Hence it appears, that if at both the Extremities of any curvilineal Plane, the Ordinates (or Perpendiculars) be ſuppoſed Nothing, we ſhall have $\overline{4Bn+2Cp+4De+2Ef+4Fg+\&c.}\times\frac{AB}{3}$, a general Expreſſion for determining, with any odd Number of Ordinates, the true Meaſure, *nearly*, of any curvilineal Plane, bounded by one Right-line and a Curve, or wholly by a Curve. The Expreſſion, in Words, will be thus: *To four times the Sum of the 1ſt, 3d, 5th,* &c. *Perpendiculars, add Double the Sum of all the Reſt; this Total being multiplied by ⅓d of the common Diſtance of the Perpendiculars, the Product gives the Area ſought,* nearly.

SCHOLIUM.

And from the ſaid Demonſtration, I have deduced two *general Rules*; *one* for approximating, with any even Number of equidiſtant perpendicular Ordinates, the Area of any curvilineal Space, bounded by two perpendicular Right-lines and a Curve; and the *other* for obtaining, with any odd Number of equidiſtant perpendicular Ordinates (or Diameters), the *true* Area, *very nearly*, of any curvilineal Space, comprehended either by a Right-line, and a Curve, or wholly by a Curve: But it is to be obſerved, that theſe Rules, for the moſt Part, will not approximate the Areas, of ſuch Planes as occur in the Subject of Gauging, ſo near as the following *general Rule*; which determines, *very nearly*, the *true* Meaſure of any curvilineal Plane, bounded by three perpendicular Right-lines and a Curve, of any Kind; or by two parallel Lines, and two Curves, of any Kind.

It may be proper to obſerve here, that it will always be neceſſary, according to our Method of conſidering the Matter, to take an *odd* Number of Ordinates; which indeed is unavoidable, when an Ordinate is taken (as it always is, in the practical Method of taking the Dimenſions of a curvilineal Back,

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SCHOLIUM.

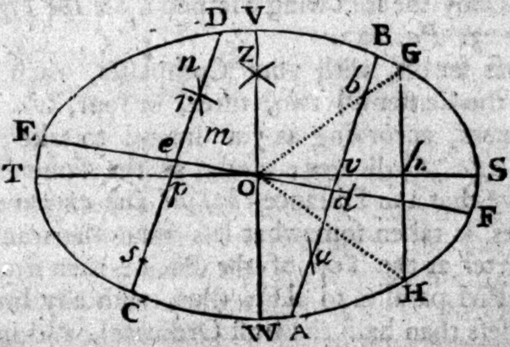
It is to be obſerved, that neither of the preceding Rules will approximate the Area ſo near as that deduced from *Cor.* 1. for the Error increaſes towards the leaſt Ordinates, on Account of their Obliquity to the Curve; the Length whereof, between any two adjacent equidiſtant Perpendiculars, keeps continually increaſing, and conſequently gives greater Latitude for different Curves to paſs through the ſame three Points, in thoſe equidiſtant Perpendiculars.

If the Number of equidiſtant Ordinates (or Perpendiculars) exceeds three; then the general Rule derived from the 1. *Cor.* will not *ſtrictly* agree with thoſe Rules obtained from the general Method of Differences; which determines a parabolic Curve of *ſome* Kind to paſs through *any* aſſigned Number of Points: Whereas, in the Method here laid down, common parabolic Curves are ſuppoſed to be deſcribed through the 1ſt, 2d, and 3d, the 3d, 4th, and 5th, and through the 5th, 6th, and 7th, *&c.* Ordinates; but the Difference ariſing from computing the Area of any curvilineal Plane, by theſe two Methods, is extremely ſmall, and can never be of the leaſt Conſequence in the Buſineſs of Gauging, *&c.*

Back, *&c.) exactly* in the Middle of the two extreme Ordinates: And therefore this Method, which is comprehended in *one general Rule*, let the *odd* Number of Ordinates be what it will, claims the Preference in Point of Expedition; ſince it will appear, by the following Examples, that a large Number of Ordinates does but very little increaſe the Labour of Computation, which is far from being the Caſe in any Rules (founded on indubitable Principles) I have hitherto met with.

Before we proceed to exemplify this Method, it may not be amiſs to give a few general Directions, for taking the Dimenſions of curvilineal Backs; ſuch as are now frequently made Uſe of by *Diſtillers*, *&c.*

Let the Bottom of an elliptical Veſſel be repreſented by the following Figure; then in Order to find the Center, and alſo to draw the tranſverſe and conjugate Diameters thereof; proceed as follows,



Firſt, with your Chalk-Line, in any Part of the Back, ſtrike a Line AB; then, with one Foot of your Compaſſes on *s*, at ſome convenient Diſtance from AB, deſcribe an Arch of a Circle, ſo as to cut that Line, as at *a*; and with the ſame Extent upon *b*, ſome Diſtance from *a* (in the Line AB) deſcribe the Arch *mn*; then upon *s*, as a Center, with the Extent *ab*, deſcribe another Arch, cutting *mn* in *r*; through the Points *s* and *r*, ſtrike (with your Chalk-Line) the Right-line CD, which is parallel to AB, and through *e* and *d*, the Middle of CD and AB, draw the Line EF; the Middle of which is (at O) the Center of the Ellipſis: Upon O, as a Center, with a Line (or String) of a ſuitable Length, let two Marks be made in the Periphery of the Figure, as at G and H, and mark the Right-line GH, which biſect (*i. e.* divide into two equal Parts) in the Point *b*; through which, and the Center O, ſtrike (with the Chalk-Line) the tranſverſe Diameter TS; and upon two Points *p* and *v*, equally diſtant from O, with an Interval greater than O*v* (or O*p*) deſcribe two Arches interſecting each other in *z*; then through O and *z*, draw the conjugate Diameter VW.

After the Bottom of the Veſſel is thus quartered, draw the Ordinates, or Right-lines, perpendicular to EF, by the following Method: *See the Figure to Exam.* 3. *Pa.* 227.

Firſt ſet off, with your Compaſſes, each Way from the Center O, two, three, or four, &c. equal Diſtances, according as you intend to take 5, 7, or 9, &c. Ordinates; and, at the ſame Time, obſerve that the Diſtance *(bc)* of the extreme Ordinates is taken ſomewhat leſs than the tranſverſe Diameter at the Top of the Back; then draw the Line PM parallel to EF; thus, with any Interval *f*O (leſs than half the leaſt Ordinate), and upon a convenient Point *m*, as a Center, deſcribe the Arch *vw*, and with the Extent *m*O, upon *f*, as a Center, deſcribe another Arch, cutting the former Arch in *n*; through the Points *n* and *f*, mark (as before) the Line PM: Then ſet off, each Way from *f*, the ſame Number of equal Diſtances, and with the ſame Extent as thoſe ſet off in the tranſverſe Diameter; that is, let *bd*, *re*, *d*O, *ef*, O*g*, *&c.* be all equal to each other; then, with the Chalk-Line, through the Points *b*, *r*; *d*, *e*; *g*, *h*; and *c*, *k*; mark the Ordinates AB, CD, GH, and IK.

Now, in Order to draw Lines from the Extremities of the Ordinates, *&c.* up the Sides of the Back (ſo that Ordinates may be taken, in any Part of its Depth, at the ſame equal Diſtances as thoſe at the Bottom) proceed in the following Manner: Hold a *(chalked)* Plumb-Line at the Top of the Back, directly over the Ordinate AB; then, order your Aſſiſtant to hold the Plummet at the Extremity A; ſtrike a Line againſt the Side of the Back, from the Bottom to the Top: Proeeed in the very ſame Manner at the other Points C, R, G, I, F, K, *&c.* to E.

A GENERAL RULE, *for determining*, very nearly, *with any odd Number of equidiſtant Perpendicular Ordinates*, &c. *the Meaſure of any curvilineal Plane, bounded at each End by an Ordinate.*

To the Sum of the firſt and laſt, or the two extreme Ordinates, add four times the Sum of the 2d, 4th, 6th, and 8th, *&c.* Ordinates, and alſo Double the Sum of all the Reſt; this Total being multiplied by one-third of the common Diſtance of the Ordinates; the Product will give the required Area, *very nearly:* Which being divided by 282, 231, and by 2150, the Quotients will be the Areas in Ale and Wine Gallons, and Malt Buſhels reſpectively.

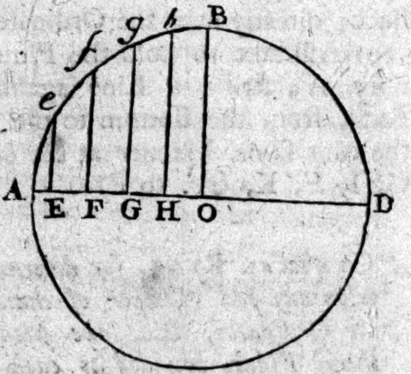
EXAMPLE I.

Let it be required to find the Area of a Circle, whoſe Diameter is 36 Inches.

In the Quadrant AOB, draw four equidiſtant Lines, perpendicular to the Diameter AD, and let their common Diſtance EF (FG, *&c.*) be 4 Inches; then, by the Property of the Circle, we get the Lengths of thoſe Perpendiculars, *viz.*

E*e* equal to 8.24
F*f* = 13.41
G*g* = 16.12
H*h* = 17.55
OB = 18.

Hence the following



OPERATION.

The Sum of the extreme Ordinates is 26.24
Four times the Sum of the 2d and 4th (30.96) is 123.84
Twice the Middle Ordinate (16.12) is 32.24

Total 182.32.

Which being multiplied by 4, and the Product divided by 3; or, which comes to the ſame, multiplied by one-third of the common Diſtance, gives the Area of the Space OB*e*E 243.093

Add the Segment AE*e*; which (by Reaſon of its Smallneſs, with Regard to the whole Circle) differs but little from a Semi-parabola . . . . } 10.98

Gives the Area of the Quadrant AOB 254.073
Which being multiplied by . . . 4

Gives the Area of the Circle 1016.292; this being divided by 282, gives 3.603 Ale Gallons; and divided by 231, gives 4.399 Wine Gallons.

The Area of the above Circle, found by the common Method, will be 3.609 Ale Gallons, and 4.406 Wine Gallons; which exceed the former but about 6 thouſandth Part of an Ale Gallon, and 7 thouſandths of a Wine Gallon: And by making Uſe of a greater Number of Ordinates, the above Differences would ſtill have been leſs.

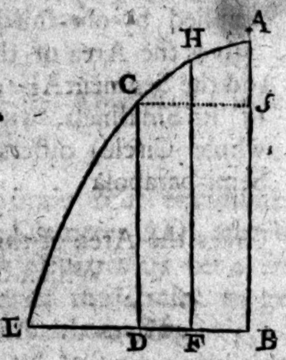
EXAMPLE I.

In a common Parabola, whoſe Abſciſſa AB is 32, and the Semi-ordinate EB 24, it is required to find the Area of the Part ABDC; when BD (or the Semi-ordinate C*s*) is equal to 12.

OPERATION.

Firſt there is given AB = 32; and, by the Property of the Curve, *Pa.* 65, we have FH = 30, and alſo CD = 24; whence, by the preceding *general Rule,*

The Sum of the two Extremes	56
Four times the Middle Perpendicular FH	120
Total	176
Multiplied by ⅓d of the common Diſtance DF (FB)	2
Gives the Area (*ſtrictly true*) of the Part ABDC	352.



For, to the parabolic Area CA*s* (*ſee Pa.* 95) 64
Add the Area of the Rectangle DC*s*B 288

Gives the required Area 352, [*as before.*

EXAMPLE 3.

To find the Area of the curvilineal Plane ERFLE, in Ale and Wine Gallons; whoſe Axis EF (biſected by RL) is ſuppoſed equal to 112 Inches, and alſo the perpendicular Ordinates, and their common Diſtance aſunder, as below.

Inches.

Ordinates
AB = 70
CD = 79
RL = 80
GH = 78.6
IK = 69.0

Their common Diſtance (*bd*, &*c.*) is 24 Inches; and therefore E*b* (or F*c*) is 8 Inches.

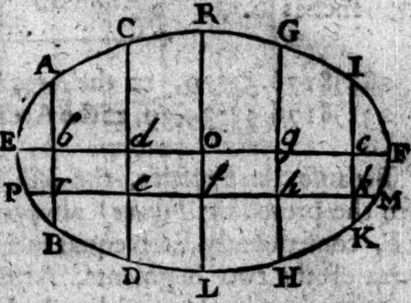
OPERATION.

OPERATION.

By the preceding *general Rule*, the Sum of the two

extreme Ordinates	139
Four times the Sum of the 2d and 4th	630.4
Double the Sum of the reſt (*viz.* the 3d)	160
Total	929.4
Multiplied by $\frac{1}{3}$ of 24 =	8
Gives the Area of the Part contained between the extreme Ordinates	7435.2

Then to this Area, add that of the Segments AEB and IFK (which, on Account of the Smallneſs of E*b* or F*c*, with Regard to AB or IK, is immaterial whether they are conſidered as Parabolas, or the Segments of Circles), and we ſhall then have the Area in Inches, of the whole Figure ERFLE.



AB = 70
Multiply by E*b* 8

Product 560

$\frac{2}{3}$ds whereof is 373.33, the Area of the Segment AEB.

Again, IK = 69
Multiply by F*c* 8

Product 552

$\frac{2}{3}$ds of which is 368, the Area of the Segment IFK.
Add 373·33

The Area of both Segments taken as Parabolas . . . } 741·33
Add 7435.2

Whole Content in Inches . . } 8176.53

282)8176.53(29, = the Area in Ale Gallons.
231)8176.53(35·39, = the Area in Wine Gallons,

Becauſe, in Practice, the Middle Ordinate RL (ſee the preceding *Figure)* always biſects the Axis EF, conſequently E*b* is equal to F*c:* Therefore the Area of the two Segments, APB and IMK, may be more readily obtained by multiplying the Sum of the two extreme Ordinates, AB and IK, by E*b* or F*c*, and taking two-thirds of the Product; or, which is the ſame Thing, multiplying the ſaid Sum of the Ordinates by twice E*b* (or twice F*c)*, and taking one-third of the Product; as in the following Operations.

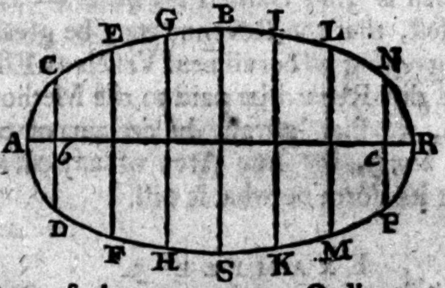
EXAMPLE 4.

Required the Area of the curvilineal Plane ABRS, in Ale and Wine Gallons, ſuppoſing AR (biſected by BS) equal to 102 Inches, the perpendicular Ordinates, and their common Diſtance as follow.

Perpendicular Ordinates
CD = 54
EF = 60
GH = 62.2
BS = 63
IK = 62.4
LM = 60.1
NP = 54.6

Their common Distance 15 Inches; and therefore A*b* (or R*c*) equal to 6 Inches.

OPERATION.



The Sum of the two extreme Ordinates 108.6
Four times the Sum of the 2d, 4th, and 6th Ordinates 732.4
Twice the Sum of the rest (*viz.* 3d and 5th) 249.2

Total 1090.2
Multiply by one-third of 15, *viz.* . . . 5

The Area of the Part DCNP 5451.0

FOR THE AREA OF THE TWO SEGMENTS.

The Sum of the first and last Ordinates 108.6
Multiply by twice A*b* (or twice R*c*) . . 12

Product 1303.2

One-third whereof is 434.4
Add the Area found above 5451

The whole Content in Inches 5885.4

282)

282)5885.4(20.87 = the Area in Ale Gallons.
231)5885.4(25.477 Wine Gallons.

The Area of the foregoing Figure being computed by the Method laid down in *Shirtcliffe*'s Gauging, *Pa.* 187, will come out 20.862 and 25.468 Ale and Wine Gallons reſpectively.

But if the Area of the ſaid Figure be computed as an Ellipſis, it will come out 21.84 Wine Gallons; which is 3.63 Gallons *too little.* — Whence it is manifeſt, that the Revenue may be greatly injured, by gauging *all* curvilineal Veſſels as Ellipſes: But, if a due Regard be paid to the Method here laid down, we ſhall always be certain of obtaining, *very nearly*, the *true* Area of any curvilineal Veſſel, let its Form be what it will.

EXAMPLE 5.

Wherein it is propoſed to find the Area of the curvilineal Space ABCDA (not *Elliptical*, but of an *unknown* Form), whoſe Dimenſions were obtained, by actual Menſuration, in the following Manner, *viz.*

The Axis AC (or twice A*a*) is equal to 151.7 Inches.

Perpendicular Ordinates,

See Overley's Gauging, Pa. 281.

EE equal to 61.4
FF = 101.2
GG = 118.0
HH = 123.7
BD = 125.2
II = 124.0
KK = 118.6
LL = 99.9
MM = 57.0

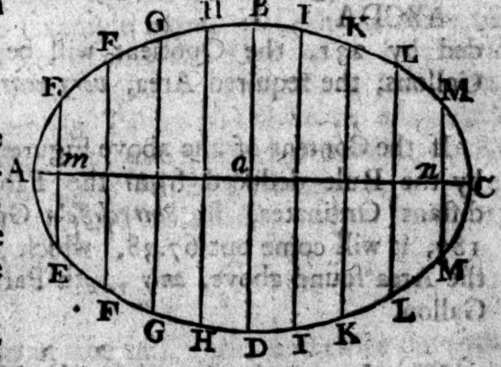
Their common Diſtance is 17 Inches; and conſequently A*m* (or C*n*) is 7.85 Inches.

OPERATION.

OPERATION.

By the foregoing *general Rule*, we have the Sum of the two extreme Ordinates . .	118.4
Four times the Sum of the 2d, 4th, 6th, and 8th Ordinates (beginning at either End) . . .	1795.2
Twice the Sum of all the reſt (*viz.* 3d, 5th, and 7th	723.6
Total	2637.2
Which being multiplied by 17, and the Product divided by 3 (or multiplied by $\frac{1}{3}$ of 17) becomes .	14944.13=

the Area in Inches, of the Space contained between the extreme Ordinates; to this, add the Area of the two Segments EAE and MCM (conſidered as Parabolas), and we ſhall then obtain the Area of the whole curvilineal Space ABCD, *very nearly*: See the Remainder of the Operation.



The Sum of the two extreme Ordinates } 118.4
Multiply by A*m* (or C*n*) 7.85

5920
9472
8288

Product 929.440

$\frac{2}{3}$ds whereof is 619.626 = the Area of both [Segments.

Add the Area of the Space found above . . } 14944.13

The Area of the whole Figure ABCDA } 15563.756; which being divided by 231, the Quotient will be 67.37 Wine Gallons, the required Area, *very nearly*.

If the Content of the above Figure be computed by the Rule deduced from the Tables of equidiſtant Ordinates, in *Shirtcliffe*'s Gauging, *Pa.* 187, it will come out 67.38, which differs from the Area found above, *only* $\frac{1}{100}$th Part of a Wine Gallon.

Note. It may be proper to take Notice, that, according to this Method of Computation, it will be the moſt commodious to write down the Dimenſions, *&c.* of a curvilineal Back, in the following Manner.

Mr. ——— 5th Back, gauged *Nov.* 22, 1764.

ORDINATES 13 Inches equidiſtant.										
In-ches.	Tranſ-verſe.	1	2	3	Conjugate 4	5	6	7	Areas.	Gallons
13	84.8	29.3	50.7	57.5	58.4	56.8	49.7	24.7	17.74	230.62
11	86.0	31.5	51.6	58.1	59.0	57.5	50.8	27.0	18.22	200.42
11	86.9	33.0	52.3	58.8	59.7	58.3	52.0	29.2	18.66	205.26
11	87.9	34.5	53.1	59.3	60.3	58.9	53.0	31.5	19.10	210.10
10	88.8	36.0	53.8	60.0	60.8	59.6	53.8	33.7	19.51	195.10
1.3	*Drip*	. . . .								20.

57.3 *Depth.*

Content 1061.50

Depth 57.3 Gall.
Drip 1.3 = 20

Neat Depth 56.0

Before I quit this Subject, it may not be amiſs to give one Example more, in Order to ſhew the Method of computing the Area of a curvilineal Plane, by a Rule (for 13 equidiſtant Ordinates) deduced from the Tables laid down in *Shirtcliffe's* Gauging, *Pa.* 187; and then to give the Operation by the *general Rule*, *Pa.* 223; whereby, I apprehend, its Utility will manifeſtly appear to every impartial Reader.

RULE.

The Sum of the 1ſt, 13th, and twice the (7th) Middle Ordinate, multiplied by 41; the Sum of the 2d, 6th, 8th, and 12th Ordinates multiplied by 216; the Sum of the 3d, 5th, 9th, and 11th Ordinates multiplied by 27; and the Sum of the 4th and 10th multiplied by 272: Then the Sum of theſe four Products being multiplied by the Diſtance of the two extreme Ordinates, and that Product divided by 1680, the Quotient will be the Area in Inches of any curvilineal Space contained between the extreme Perpendiculars.

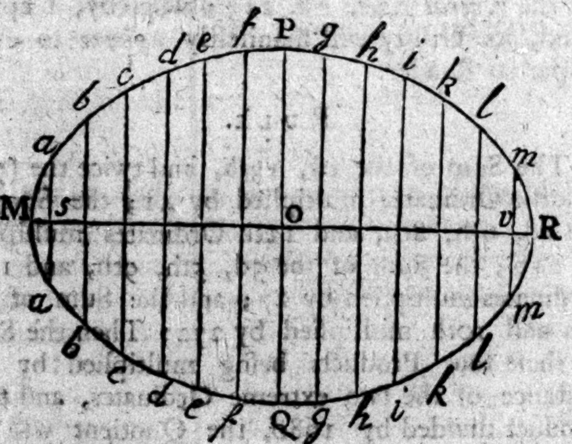
EXAMPLE.

To find the Area of the curvilineal Plane MPRQ, in Wine Gallons, when MR (or twice MO) is equal to 148 Inches; the perpendicular Ordinates, and their common Diſtance as below.

Perpendicular Ordinates

aa = 82.4
bb = 96.5
cc = 112.0
dd = 119.2
ee = 121.3
ff = 122.4
PQ = 123.0
gg = 122.6
hh = 121.2
ii = 118.9
kk = 112.0
ll = 96.3
mm = 82.4

Their common Diſtance is 12 Inches; and therefore M*s* (or R*v*) is 2 Inches.



OPERATION, *by the preceding Rule.*

The 1ſt, 13th, and twice the Middle Ordinate { 82.4
82.4
246.0

Sum 410.8
Multiplied by 41

4108
16432

1ſt Product 16842.8

The 2d, 6th, 8th, and 12th Ordinates { 96.5
122.4
122.6
96.3

Sum 437.8
Multiplied by 216

26268
4378
8756

2d Product 94564.8

The

The 3d, 5th, 9th, and 11th Ordinates	112.0 121.3 121.2 112.0
Sum	466.5
	27
	32655
	9330
3d Product	12595.5
The 4th and 10th Ordinates	119.2 118.9
Sum	238.1
Multiplied by	272
	4762
	16667
	4762
4th Product	64763.2
3d Product	12595.5
2d Product	94564.8
1ſt Product	16842.8
The Sum of the four Products	188766.3
Multiplied by the Diſtance of the extreme Ordinates	. . . 144
	7550652
	7550652
	1887663
Gives	27182347.2
	1680)

1680)27182347.20(16179.96
1680

10382
10080

3023
1680

13434
11760

16747
15120

16272
15120

11520
10080

1440

219.73 = the Area of the 2 [Seg. taken as Parab.
16399.69 = the whole Con- [tent in Inches.

231)16399.69(70.99 Wine Gallons; the required Area, *nearly*.

OPERATION, by the *general Rule*, *Pa.* 223.

The Sum of the extreme Ordinates, 164.8.

$$\text{The 2d, 4th, 6th, 8th, 10th, and 12th Ordinates}\left\{\begin{array}{r}96.5\\119.2\\122.4\\122.6\\118.9\\96.3\end{array}\right.$$

Sum 675.9 multiplied by 4, gives [2703.6.

The 3d, 5th, 7th, 9th, and 11th Ordinates $\left\{\begin{array}{r} 112.0 \\ 121.3 \\ 123.0 \\ 121.2 \\ 112.0 \end{array}\right.$

Sum 589.5

Which multiplied by 2, gives 1179.0, to which

164.8 and 2703.6 being added, gives . . . 4047.4

Multiply by $\frac{1}{3}$ of 12 . . . 4

16189.6

The Area of the Segments 219.73

231)16409.33(71.03 Wine Gallons.

The Areas obtained, by theſe two Methods, differ but about $\frac{4}{100}$th Part of a Wine Gallon; and both the Operations were given at Length, in Order to ſhew the vaſt Advantage which the latter has over the former.

☞ It is manifeſt (by the Writers on Fluxions) that the Radius of Curvature of a *Parabola* is infinite, at an infinite Diſtance from its Vertex; therefore the Difference between a Right-line and a parabolic Curve (of any Kind), at an infinite Diſtance from the Vertex, is leſs than any aſſignable Quantity; and, conſequently, as the Methods here propoſed (or thoſe Rules derived from the general Principles) are *ſtrictly true*, in *every* Part of a *Parabolic Curve*, it evidently follows, that, at an infinite Diſtance from the Vertex, the *Meaſure* of a Space, obtained by any of theſe Rules, will differ from *that*, when conſidered as a right-lined Space, by a Quantity leſs than any given, or aſſignable, Quantity whatever: But (by *Lemma* 1ſt, *Pa*. 99, of *De L'Hoſpital's* Conic Sections) *if the Difference of two Quantities does continually diminiſh, ſo that at laſt it becomes leſs than any given Quantity; then will thoſe Quantities at laſt be equal.* — Hence it is evident, that by theſe Methods we can obtain the *true* (and not the *approximate*) Meaſure of any rectilineal Plane whatever: For any right-lined Plane may be divided into Quadrilaterals (whereof two Sides *muſt* be parallel) and Triangles; then may the *true* Meaſures of thoſe Figures be ſeparately found, by any aſſigned Number of equidiſtant Perpendiculars whatever. — Or the *true* Area may be obtained, by dividing the Figure into Triangles.

Suppoſe, for Example, it was required to find the Meaſure of the right-lined Space ABED, when AB (*Fig.* XVII. is parallel to DE) = 20, DE = 12, and the Perpendicular DF = 30 Inches.

First, *by three equidiſtant Perpendiculars.*

The Sum of the Extremes 32
Four times the Middle Perpendicular (16) = 64

Total 96
Multiply by ⅓d of 15 . . 5

Gives the required Meaſure 480 = CC

$(\frac{AB+DE}{2}) \times DF = 16 \times 30$, as is well known from other Principles.

By five equidiſtant Perpendiculars.

If DP (*Fig.* XVII.) be drawn parallel to EB, and GG, CC and II be drawn parallel to AB, equidiſtant from one another, and the other Dimenſions the ſame as above; then it is evident that $Id=2$, $Cc=4$, and $Gb=6$; whence $DE=12$, $II=14$, $CC=16$, $GG=18$, and $AB=20$; then, by the the *general Rule, Pa.* 223.

The Sum of the two extreme Perpendiculars 32
Four times the Sum of the 2d and 4th Perpendiculars 128
Twice the Sum of the reſt (*viz.* the 3d) 32

Total 192
Multiply by ⅓d of the common Diſtance (7.5) . . 2.5

960
384

The Content as before 480.0

Suppoſe, in the Triangle ADP (*Fig.* XVII.), $AP=8$, $DF=30$; which being divided into eight equal Parts, and Lines drawn parallel to the Baſe, we ſhall have $aa=1$, $Id=2$, $nn=3$, $Cc=4$, $rr=5$, $Gb=6$, $ee=7$, and $AP=8$: Whence, by *Cor.* 2, *Pa.* 219, we get $\overline{\overline{1+3+5+7}\times 4+8+\overline{2+4+6}\times 2} \times \frac{3.75}{3} = 120$ $(= 30 \times 4) =$ the Area of the Triangle ADP.

A TABLE *of the Areas of Circles in* ALE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 120 *Inches.*

Dia. in Inc.	0	.1	.2	.3	.4
1	0.0027	0.0033	0.0040	0.0047	0.0054
2	0.0111	0.0122	0.0134	0.0147	0.0160
3	0.0250	0.0267	0.0285	0.0303	0.0321
4	0.0445	0.0468	0.0491	0.0514	0.0539
5	0.0696	0.0724	0.0753	0.0782	0.0812
6	0.1002	0.1036	0.1070	0.1105	0.1140
7	0.1364	0.1403	0.1443	0.1484	0.1525
8	0.1782	0.1827	0.1872	0.1918	0.1965
9	0.2255	0.2306	0.2357	0.2408	0.2460
10	0.2785	0.2841	0.2897	0.2954	0.3012
11	0.3369	0.3431	0.3493	0.3556	0.3619
12	0.4010	0.4077	0.4145	0.4213	0.4282
13	0.4706	0.4779	0.4852	0.4926	0.5000
14	0.5458	0.5537	0.5615	0.5695	0.5775
15	0.6266	0.6350	0.6434	0.6519	0.6605
16	0.7129	0.7219	0.7309	0.7399	0.7490
17	0.8048	0.8143	0.8239	0.8335	0.8432
18	0.9022	0.9124	0.9225	0.9327	0.9429
19	1.0054	1.0160	1.0266	1.0374	1.0482
20	1.1140	1.1252	1.1364	1.1477	1.1590
21	1.2282	1.2399	1.2517	1.2635	1.2754
22	1.3479	1.3602	1.3726	1.3850	1.3974
23	1.4733	1.4861	1.4990	1.5120	1.5250
24	1.6042	1.6176	1.6310	1.6445	1.6581
25	1.7406	1.7546	1.7686	1.7827	1.7968
26	1.8827	1.8972	1.9118	1.9264	1.9411
27	2.0303	2.0454	2.0605	2.0757	2.0909
28	2.1835	2.1991	2.2148	2.2305	2.2463
29	2.3422	2.3584	2.3746	2.3909	2.4073

A TABLE *of the Areas of Circles in* ALE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 120 *Inches.*

Dia. in Inc.	.5	.6	.7	.8	.9
1	0.0062	0.0071	0.0080	0.0090	0.0100
2	0.0174	0.0188	0.0203	0.0218	0.0234
3	0.0341	0.0360	0.0381	0.0402	0.0423
4	0.0563	0.0589	0.0615	0.0641	0.0668
5	0.0842	0.0873	0.0904	0.0936	0.0969
6	0.1176	0.1213	0.1250	0.1287	0.1325
7	0.1566	0.1608	0.1651	0.1694	0.1738
8	0.2012	0.2059	0.2108	0.2156	0.2206
9	0.2513	0.2566	0.2620	0.2674	0.2729
10	0.3070	0.3129	0.3188	0.3248	0.3308
11	0.3683	0.3747	0.3812	0.3877	0.3943
12	0.4351	0.4421	0.4492	0.4563	0.4634
13	0.5075	0.5151	0.5227	0.5303	0.5381
14	0.5855	0.5936	0.6018	0.6100	0.6183
15	0.6691	0.6777	0.6864	0.6952	0.7041
16	0.7582	0.7674	0.7767	0.7860	0.7954
17	0.8529	0.8627	0.8725	0.8824	0.8923
18	0.9532	0.9635	0.9739	0.9843	0.9948
19	1.0590	1.0699	1.0808	1.0918	1.1029
20	1.1704	1.1818	1.1933	1.2049	1.2165
21	1.2874	1.2994	1.3114	1.3235	1.3357
22	1.4099	1.4225	1.4351	1.4478	1.4605
23	1.5380	1.5511	1.5643	1.5775	1.5908
24	1.6717	1.6854	1.6991	1.7129	1.7267
25	1.8110	1.8252	1.8395	1.8538	1.8682
26	1.9558	1.9706	1.9854	2.0003	2.0153
27	2.1062	2.1215	2.1369	2.1524	2.1679
28	2.2621	2.2781	2.2940	2.3100	2.3261
29	2.4237	2.4401	2.4567	2.4732	2.4899

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	0	.1	.2	.3	.4
30	2.5065	2.5233	2.5401	2.5569	2.5738
31	2.6764	2.6937	2.7111	2.7285	2.7459
32	2.8519	2.8697	2.8877	2.9056	2.9236
33	3.0329	3.0513	3.0698	3.0883	3.1069
34	3.2195	3.2385	3.2575	3.2760	3.2957
35	3.4117	3.4312	3.4508	3.4704	3.4901
36	3.6094	3.6295	3.6497	3.6698	3.6901
37	3.8128	3.8334	3.8541	3.8748	3.8956
38	4.0216	4.0428	4.0641	4.0854	4.1067
39	4.2361	4.2578	4.2796	4.3015	4.3234
40	4.4561	4.4784	4.5008	4.5232	4.5457
41	4.6817	4.7046	4.7275	4.7505	4.7735
42	4.9129	4.9363	4.9598	4.9833	5.0069
43	5.1496	5.1736	5.1976	5.2217	5.2459
44	5.3919	5.4164	5.4410	5.4657	5.4904
45	5.6398	5.6649	5.6900	5.7152	5.7405
46	5.8932	5.9189	5.9446	5.9703	5.9962
47	6.1522	6.1784	6.2047	6.2310	6.2574
48	6.4168	6.4436	6.4704	6.4973	6.5242
49	6.6870	6.7143	6.7417	6.7691	6.7966
50	6.9627	6.9906	7.0185	7.0465	7.0745
51	7.2440	7.2724	7.3009	7.3295	7.3581
52	7.5309	7.5599	7.5889	7.6180	7.6472
53	7.8233	7.8528	7.8825	7.9121	7.9418
54	8.1213	8.1514	8.1816	8.2118	8.2421
55	8.4249	8.4555	8.4863	8.5170	8.5479
56	8.7340	8.7652	8.7965	8.8279	8.8592
57	9.0487	9.0805	9.1124	9.1442	9.1762
58	9.3690	9.4014	9.4338	9.4662	9.4987
59	9.6949	9.7278	9.7607	9.7937	9.8268
60	10.0263	10.0598	10.0933	10.1268	10.16[illegible]4

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.5	.6	.7	.8	.9
30	2.5908	2.6078	2.6249	2.6420	2.6592
31	2.7635	2.7810	2.7987	2.8164	2.8341
32	2.9417	2.9598	2.9780	2.9963	3.0146
33	3.1255	3.1442	3.1630	3.1818	3.2006
34	3.3149	3.3342	3.3535	3.3728	3.3922
35	3.5099	3.5297	3.5495	3.5694	3.5894
36	3.7104	3.7308	3.7512	3.7716	3.7922
37	3.9165	3.9374	3.9584	3.9794	4.0005
38	4.1282	4.1496	4.1712	4.1928	4.2144
39	4.3454	4.3674	4.3895	4.4117	4.4339
40	4.5682	4.5908	4.6134	4.6361	4.6589
41	4.7966	4.8197	4.8429	4.8662	4.8895
42	5.0305	5.0542	5.0780	5.1018	5.1257
43	5.2701	5.2943	5.3186	5.3430	5.3674
44	5.5151	5.5400	5.5648	5.5898	5.6147
45	5.7658	5.7912	5.8166	5.8421	5.8676
46	6.0220	6.0480	6.0739	6.1000	6.1261
47	6.2838	6.3103	6.3369	6.3635	6.3901
48	6.5512	6.5782	6.6053	6.6325	6.6597
49	6.8241	6.8517	6.8794	6.9071	6.9349
50	7.1027	7.1308	7.1590	7.1873	7.2156
51	7.3867	7.4154	7.4442	7.4730	7.5019
52	7.6764	7.7057	7.7350	7.7644	7.7938
53	7.9716	8.0014	8.0313	8.0613	8.0913
54	8.2724	8.3028	8.3332	8.3637	8.3943
55	8.5788	8.6097	8.6407	8.6717	8.7029
56	8.8907	8.9222	8.9537	8.9854	9.0170
57	9.2082	9.2402	9.2724	9.3045	9.3367
58	9.5313	9.5639	9.5965	9.6293	9.6620
59	9.8599	9.8931	9.9263	9.9596	9.9929
60	10.1941	10.2278	10.2616	10.2955	10.3294

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
61	10.3633	10.39.3	10.4314	10.4655	10.4997
62	10.7059	10.7404	10.7751	10.8097	10.8445
63	11.0540	11.0891	11.1243	11.1595	11.1948
64	11.4077	11.4434	11.4791	11.5149	11.5508
65	11.7670	11.8032	11.8395	11.8759	11.9123
66	12.1318	12.1686	12.2055	12.2424	12.2793
67	12.5023	12.5396	12.5770	12.6145	12.6520
68	12.8783	12.9162	12.9541	12.9921	13.0302
69	13.2598	13.2983	13.3368	13.3754	13.4140
70	13.6469	13.6860	13.7250	13.7642	13.8034
71	14.0396	14.0792	14.1188	14.1585	14.1983
72	14.4379	14.4780	14.5182	14.5585	14.5988
73	14.8417	14.8824	14.9232	14.9640	15.0048
74	15.2512	15.2924	15.3337	15.3751	15.4165
75	15.6661	15.7079	15.7498	15.7917	15.8337
76	16.0867	16.1290	16.1715	16.2139	16.2565
77	16.5128	16.5557	16.5987	16.6417	16.6848
78	16.9445	16.9880	17.0315	17.0751	17.1187
79	17.3818	17.4258	17.4699	17.5140	17.5582
80	17.8246	17.8692	17.9138	17.9585	18.0033
81	18.2730	18.3181	18.3633	18.4086	18.4539
82	18.7270	18.7727	18.8184	18.8642	18.9101
83	19.1865	19.2328	19.2791	19.3255	19.3719
84	19.6516	19.6984	19.7453	19.7922	19.8392
85	20.1223	20.1697	20.2171	20.2646	20.3121
86	20.5985	20.6465	20.6945	20.7425	20.7906
87	21.0804	21.1289	21.1774	21.2260	21.2747
88	21.5678	21.6168	21.6659	21.7151	21.7643
89	22.0607	22.1103	22.1600	22.2097	22.2595
90	22.5593	22.6094	22.6596	22.7099	22.7602
91	23.0634	23.1141	23.1649	23.2157	23.2666

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.5	.6	.7	.8	.9
61	10.5339	10.5682	10.6025	10.6369	10.6714
62	10.8792	10.9141	10.9490	10.9839	11.0189
63	11.2302	11.2656	11.3010	11.3365	11.3721
64	11.5867	11.6226	11.6586	11.6947	11.7308
65	11.9487	11.9852	12.0218	12.0584	12.0951
66	12.3164	12.3534	12.3906	12.4277	12.4650
67	12.6896	12.7272	12.7649	12.8026	12.8404
68	13.0683	13.1065	13.1448	13.1831	13.2214
69	13.4527	13.4914	13.5302	13.5691	13.6080
70	13.8426	13.8819	13.9212	13.9607	14.0001
71	14.2381	14.2779	14.3178	14.3578	14.3978
72	14.6391	14.6795	14.7200	14.7605	14.8011
73	15.0458	15.0867	15.1277	15.1688	15.2100
74	15.4580	15.4995	15.5411	15.5827	15.6244
75	15.8757	15.9178	15.9599	16.0021	16.0444
76	16.2991	16.3417	16.3844	16.4271	16.4699
77	16.7280	16.7712	16.8144	16.8577	16.9011
78	17.1624	17.2062	17.2500	17.2939	17.3378
79	17.6025	17.6468	17.6912	17.7356	17.7801
80	18.0481	18.0930	18.1379	18.1829	18.2279
81	18.4993	18.5447	18.5902	18.6357	18.6813
82	18.9560	19.0020	19.0481	19.0941	19.1403
83	19.4184	19.4649	19.5115	19.5581	19.6049
84	19.8863	19.9334	19.9805	20.0277	20.0750
85	20.3597	20.4074	20.4551	20.5029	20.5507
86	20.8388	20.8870	20.9352	20.9836	21.0319
87	21.3234	21.3721	21.4210	21.4698	21.5188
88	21.8135	21.8629	21.9123	21.9617	22.0112
89	22.3093	22.3592	22.4091	22.4591	22.5092
90	22.8106	22.8611	22.9115	22.9621	23.0127
91	23.3175	23.3685	23.4195	23.4707	23.5218

The

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
92	23.5730	23.6243	23.6756	23.7270	23.7785
93	24.0883	24.1401	24.1920	24.2439	24.2959
94	24.6091	24.6615	24.7139	24.7664	24.8190
95	25.1355	25.1884	25.2414	25.2945	25.3476
96	25.6674	25.7209	25.7745	25.8281	25.8818
97	26.2050	26.2590	26.3131	26.3673	26.4215
98	26.7481	26.8027	26.8573	26.9121	26.9668
99	27.2967	27.3519	27.4071	27.4624	27.5177
100	27.8510	27.9067	27.9625	28.0183	28.0742
101	28.4108	28.4670	28.5234	28.5798	28.6362
102	28.9761	29.0330	29.0899	29.1468	29.2038
103	29.5471	29.6045	29 6619	29.7194	29.7770
104	30.1236	30.1815	30.2396	30.2970	30.3558
105	30.7057	30.7642	30.8228	30 8814	30.9401
106	31.2933	31.3524	31.4115	31.4707	31.5300
107	31.8866	31.9462	32.0059	32.0656	32.1254
108	32.4854	32.5455	32.6058	32.6661	32.7264
109	33.0897	33.1505	33.2113	33.2721	33.3330
110	33.6997	33.7610	33.8223	33.8837	33.9452
111	34.3152	34.3770	34.4389	34.5009	34.5629
112	34.9362	34.9987	35.0611	35.1237	35.1862
113	35.5629	35.6259	35.6889	35.7520	35.8151
114	36.1951	36.2586	36.3222	36.3859	36.4496
115	36.8329	36.8970	36.9611	37.0253	37.0896
116	37.4763	37.5409	37.6056	37.6703	37.7352
117	38.1252	38.1904	38.2556	38.3209	38.3863
118	38.7797	38.8454	38.9113	38.9771	39.0430
119	39.4398	39.5061	39.5724	39.6389	39.7053
120	40.1054	. . .	. . .	. . .	. . .

The Areas of Circles in ALE GALLONS.

Dia. in Inc.	·5	.6	·7	.8	·9
92	23.8300	23.8815	23.9331	23.9848	24.0365
93	24.3480	24.4001	24.4523	24.5045	24.5568
94	24.8716	24.9243	24.9770	25.0298	25.0826
95	25.4008	25.4540	25.5073	25.5606	25.6140
96	25 9355	25.9893	26.0431	26.0970	26.1510
97	26.4758	26.5301	26.5845	26.6390	26.6935
98	27.0217	27.0766	27.1315	27.1865	27.2416
99	27.5731	27.6280	27.6841	27.7397	27.7953
100	28.1302	28.1862	28.2422	28.2983	28.3545
101	28.6927	28.7493	28.8059	28.8626	28.9193
102	29.2609	29.3180	29.3752	29.4324	29.4897
103	29.8346	29.8923	29.9501	30.0078	30.0657
104	30.4139	30.4722	30.5305	30.5888	30.6472
105	30.9988	31.0576	31.1165	31.1754	31.2343
106	31.5893	31.6486	31.7080	31.7675	31.8270
107	32.1853	32.2452	32.3051	32.3652	32.4252
108	32.7868	32.8473	32.9078	32.9684	33.0290
109	33.3940	33.4550	33.5161	33.5772	33.6384
110	34.0067	34.0683	34.1299	34.1916	34.2534
111	34.6250	34.6871	34.7493	34.8116	34.8739
112	35.2489	35.3116	35.3743	35.4371	35.5000
113	35.8783	35.9416	36.0049	36.0682	36.1316
114	36.5133	36.5771	36.6410	36.7049	36.7689
115	37.1539	37.2182	37.2827	37.3471	37.4117
116	37.8000	37.8649	37.9299	37.9950	38.0600
117	38.4517	38.5172	38.5827	38.6483	38.7140
118	39.1090	39.1751	39.2411	39.3073	39.3735
119	39.7719	39.8385	39.9051	39.9718	40.0386

A TABLE *of the Areas of Circles in* WINE GALLONS, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 120 *Inches.*

Dia. in Inc.	.0	1.	.2	.3	.4
1.	0.0034	0.0041	0.0048	0.0057	0.0066
2	0.0136	0.0149	0.0164	0.0179	0.0195
3	0.0306	0.0326	0.0348	0.0370	0.0393
4	0.0544	0.0571	0.0599	0.0628	0.0658
5	0.0850	0.0884	0.0919	0.0955	0.0991
6.	0.1224	0.1265	0.1306	0.1349	0.1392
7	0.1666	0.1713	0.1762	0.1811	0.1861
8.	0.2176	0.2230	0.2286	0.2342	0.2399
9	0.2754	0.2815	0.2877	0.2940	0.3004
10	0.3400	0.3468	0.3537	0.3607	0.3677
11	0.4114	0.4189	0.4264	0.4341	0.4418
12	0.4896	0.4977	0.5060	0.5143	0.5227
13	0.5746	0.5834	0.5924	0.6014	0.6105
14	0.6664	0.6759	0.6855	0.6952	0.7050
15	0.7650	0.7752	0.7855	0.7959	0.8063
16	0.8704	0.8813	0.8922	0.9033	0.9144
17	0.9826	0.9941	1.0058	1.0175	1.0293
18	1.1016	1.1138	1.1262	1.1386	1.1511
19	1.2274	1.2403	1.2533	1.2664	1.2796
20	1.3600	1.3736	1.3873	1.4011	1.4149
21	1.4994	1.5137	1.5280	1.5425	1.5570
22	1.6456	1.6605	1.6756	1.6907	1.7059
23	1.7980	1.8142	1.8300	1.8458	1.8617
24	1.9584	1.9747	1.9911	2.0076	2.0242
25	2.1250	2.1420	2.1591	2.1763	2.1935
26	2.2984	2.3161	2.3338	2.3517	2.3696
27	2.4786	2.4969	2.5154	2.5339	2.5525
28	2.6656	2.6846	2.7038	2.7230	2.7423
29	2.8594	2.8791	2.8989	2.9188	2.9388

A

A TABLE *of the Areas of Circles in* Wine Gallons, *to all Diameters in Inches and Inches and Tenths, from* 1 *to* 120 *Inches.*

Dia. in Inc.	·5	.6	·7	.8	·9
1	0.0076	0.0087	0.0098	0.0110	0.0122
2	0.0212	0.0229	0.0247	0.0266	0.0285
3	0.0416	0.0440	0.0465	0.0490	0.0517
4	0.0688	0.0719	0.0751	0.0783	0.0816
5	0.1028	0.1066	0.1104	0.1143	0.1183
6	0.1436	0.1481	0.1526	0.1572	0.1618
7	0.1912	0.1963	0.2015	0.2068	0.2121
8	0.2456	0.2514	0.2573	0.2632	0.2693
9	0.3068	0.3133	0.3199	0.3265	0.3332
10	0.3748	0.3820	0.3892	0.3965	0.4039
11	0.4496	0.4575	0.4654	0.4734	0.4814
12	0.5312	0.5397	0.5483	0.5570	0.5657
13	0.6196	0.6288	0.6381	0.6474	0.6569
14	0.7148	0.7247	0.7347	0.7447	0.7548
15	0.8168	0.8274	0.8380	0.8487	0.8595
16	0.9256	0.9369	0.9482	0.9596	0.9710
17	1.0412	1.0531	1.0651	1.0772	1.0893
18	1.1636	1.1762	1.1889	1.2016	1.2145
19	1.2928	1.3061	1.3195	1.3329	1.3464
20	1.4288	1.4428	1.4568	1.4709	1.4851
21	1.5716	1.5863	1.6010	1.6158	1.6306
22	1.7212	1.7365	1.7519	1.7674	1.7829
23	1.8776	1.8936	1.9097	1.9258	1.9421
24	2.0408	2.0575	2.0743	2.0911	2.1080
25	2.2108	2.2282	2.2456	2.2631	2.2807
26	2.3876	2.4057	2.4238	2.4420	2.4602
27	2.5712	2.5899	2.6087	2.6276	2.6465
28	2.7616	2.7810	2.8005	2.8200	2.8397
29	2.9588	2.9789	2.9991	3.0193	3.0396

The Areas of Circles in Wine Gallons.

Dia. in Inc.	.0	.1	.2	.3	.4
30	3.0600	3.0804	3.1009	3.1215	3.1421
31	3.2674	3.2885	3.3096	3.3309	3.3522
32	3.4816	3.5033	3.5252	3.5471	3.5691
33	3.7026	3.7250	3.7476	3.7702	3.7929
34	3.9304	3.9535	3.9767	4.0000	4.0234
35	4.1650	4.1888	4.2127	4.2367	4.2607
36	4.4064	4.4309	4.4554	4.4801	4.5048
37	4.6546	4.6797	4.7050	4.7303	4.7557
38	4.9096	4.9354	4.9614	4.9874	5.0135
39	5.1714	5.1979	5.2245	5.2512	5.2780
40	5.4400	5.4672	5.4945	5.5219	5.5493
41	5.7154	5.7433	5.7712	5.7993	5.8274
42	5.9976	6.0261	6.0548	6.0835	6.1123
43	6.2866	6.3158	6.3452	6.3746	6.4041
44	6.5824	6.6123	6.6423	6.6724	6.7026
45	6.8850	6.9156	6.9463	6.9771	7.0079
46	7.1944	7.2257	7.2570	7.2885	7.3200
47	7.5106	7.5425	7.5746	7.6067	7.6389
48	7.8336	7.8662	7.8990	7.9318	7.9647
49	8.1634	8.1967	8.2301	8.2636	8.2972
50	8.5000	8.5340	8.5681	8.6023	8.6365
51	8.8434	8.8781	8.9128	8.9477	8.9826
52	9.1936	9.2289	9.2644	9.2999	9.3355
53	9.5506	9.5866	9.6228	9.6590	9.6953
54	9.9144	9.9511	9.9879	10.0248	10.0618
55	10.2850	10.3224	10.3599	10.3975	10.4351
56	10.6624	10.7005	10.7386	10.7769	10.8152
57	11.0466	11.0853	11.1242	11.1631	11.2021
58	11.4376	11.4770	11.5166	11.5562	11.5959
59	11.8354	11.8755	11.9157	11.9560	11.9964
60	12.2400	12.2808	12.3217	12.3627	12.4037

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	·5	.6	·7	.8	·9
30	3.1628	3.1836	3.2044	3.2253	3.2463
31	3.3736	3.3951	3.4166	3.4382	3.4598
32	3.5912	3.6133	3.6355	3.6578	3.6801
33	3.8156	3.8384	3.8613	3.8842	3.9073
34	4.0468	4.0703	4.0939	4.1175	4.1412
35	4.2848	4.3090	4.3332	4.3575	4.3819
36	4.5296	4.5545	4.5794	4.6044	4.6294
37	4.7812	4.8067	4.8323	4.8580	4.8837
38	5.0396	5.0658	5.0921	5.1184	5.1449
39	5.3048	5.3317	5.3587	5.3857	5.4128
40	5.5768	5.6044	5.6320	5.6597	5.6875
41	5.8556	5.8839	5.9122	5.9406	5.9690
42	6.1412	6.1701	6.1991	6.2282	6.2573
43	6.4336	6.4632	6.4929	6.5226	6.5525
44	6.7328	6.7631	6.7935	6.8239	6.8544
45	7.0388	7.0698	7.1008	7.1319	7.1631
46	7.3516	7.3833	7.4150	7.4468	7.4786
47	7.6712	7.7035	7.7359	7.7684	7.8009
48	7.9976	8.0306	8.0637	8.0968	8.1301
49	8.3308	8.3645	8.3983	8.4321	8.4660
50	8.6708	8.7052	8.7396	8.7741	8.8087
51	9.0176	9.0527	9.0878	9.1230	9.1582
52	9.3712	9.4069	9.4427	9.4786	9.5145
53	9.7316	9.7680	9.8045	9.8410	9.8777
54	10.0988	10.1359	10.1731	10.2103	10.2476
55	10.4728	10.5106	10.5484	10.5863	10.6243
56	10.8536	10.8921	10.9306	10.9692	11.0078
57	11.2412	11.2803	11.3195	11.3588	11.3981
58	11.6356	11.6754	11.7153	11.7552	11.7953
59	12.0368	12.0773	12.1179	12.1585	12.1992
60	12.4448	12.4860	12.5272	12.5685	12.6099

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
61	12.6514	12.6929	12.7344	12.7761	12.8178
62	13.0696	13.1117	13.1540	13.1963	13.2387
63	13.4946	13.5374	13.5804	13.6234	13.6665
64	13.9264	13.9699	14.0135	14.0572	14.1010
65	14.3650	14.4092	14.4535	14.4979	14.5423
66	14.8104	14.8553	14.9002	14.9453	14.9904
67	15.2626	15.3081	15.3538	15.3995	15.4453
68	15.7216	15.7678	15.8142	15.8606	15.9071
69	16.1874	16.2343	16.2813	16.3284	16.3756
70	16.6600	16.7076	16.7553	16.8031	16.8509
71	17.1394	17.1877	17.2360	17.2845	17.3330
72	17.6256	17.6745	17.7236	17.7727	17.8219
73	18.1186	18.1682	18.2180	18.2678	18.3177
74	18.6184	18.6687	18.7191	18.7696	18.8202
75	19.1250	19.1760	19 2271	19.2783	19.3295
76	19.6384	19.6901	19.7418	19.7937	19.8456
77	20.1586	20.2109	20.2634	20.3159	20.3685
78	20.6856	20.7386	20.7918	20.8450	20.8983
79	21.2194	21.2731	21.3269	21.3808	21 4348
80	21.7600	21.8144	21.8689	21.9235	21.9781
81	22.3074	22.3625	22.4176	22.4729	22.5282
82	22.8616	22.9173	22.9732	23.0291	23.0851
83	23.4226	23.4790	23.5356	23.5922	23.6489
84	23.9904	24 0475	24.1047	24.1620	24.2194
85	24.5650	24.6228	24.6807	24.7387	24.7967
86	25.1464	25.2049	25.2634	25.3221	25.3808
87	25.7346	25.7937	25.8530	25.9123	25.9717
88	26.3296	26 3894	26.4494	26.5094	26.5695
89	26.9314	26.9919	27.0525	27.1132	27.1740
90	27.5400	27.6012	27.6625	27.7239	27.7853
91	28.1554	28.2173	28.2792	28.3413	28.4034

The

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	·5	.6	·7	.8	·9
61	12.8596	12.9015	12.9434	12.9854	13.0274
62	13.2812	13.3237	13.3663	13.4090	13.4517
63	13.7096	13.7528	13.7961	13.8394	13.8829
64	14.1448	14.1887	14.2327	14.2767	14.3208
65	14.5868	14.6314	14.6760	14.7207	14.7655
66	15.0356	15.0809	15.1262	15.1716	15.2170
67	15.4912	15.5371	15.5831	15.6292	15.6753
68	15.9536	16.0002	16.0469	16.0936	16.1405
69	16.4228	16.4701	16.5175	16.5649	16.6124
70	16.8988	16.9468	16.9948	17.0429	17.0911
71	17.3816	17.4303	17.4790	17.5278	17.5766
72	17.8712	17.9205	17.9699	18.0194	18.0689
73	18.3676	18.4176	18.4677	18.5178	18.5681
74	18.8708	18.9215	18.9723	19.0231	19.0740
75	19.3808	19.4322	19.4836	19.5351	19.5867
76	19.8976	19.9497	20.0018	20.0540	20.1062
77	20.4212	20.4739	20.5267	20.5796	20.6325
78	20.9516	21.0050	21.0585	21.1120	21.1657
79	21.4888	21.5429	21.5971	21.6513	21.7056
80	22.0328	22.0876	22.1424	22.1973	22.2523
81	22.5836	22.6391	22.6946	22.7502	22.8058
82	23.1412	23.1973	23.2535	23.3098	23.3661
83	23.7056	23.7624	23.8193	23.8762	23.9333
84	24.2768	24.3343	24.3919	24.4495	24.5072
85	24.8548	24.9130	24.9712	25.0295	25.0879
86	25.4396	25.4985	25.5574	25.6164	25.6754
87	26.0312	26.0907	26.1503	26.2100	26.2697
88	26.6296	26.6898	26.7501	26.8104	26.8709
89	27.2348	27.2957	27.3567	27.4177	27.4788
90	27.8468	27.9084	27.9700	28.0317	28.0935
91	28.4656	28.5279	28.5902	28.6526	28.7150

The Areas of Circles in WINE GALLONS.

Dia. in Inc.	.0	.1	.2	.3	.4
92	28.7776	28.8401	28.9028	28.9655	29.0283
93	29.4066	29.4698	29.5332	29.5966	29.6601
94	30.0424	30.1063	30.1703	30.2344	30.2986
95	30.6850	30.7496	30.8143	30.8791	30.9439
96	31.3344	31.3997	31.4650	31.5305	31.5960
97	31.9906	32.0565	32.1226	32.1887	32.2549
98	32.6536	32.7202	32.7870	32.8538	32.9207
99	33.3234	33.3907	33.4581	33.5256	33.5932
100	34.0000	34.0680	34.1361	34.2043	34.2725
101	34.6834	34.7521	34.8208	34.8897	34.9586
102	35.3736	35.4429	35.5124	35.5819	35.6515
103	36.0706	36.1406	36.2108	36.2810	36.3513
104	36.7744	36.8451	36.9159	36.9808	37.0578
105	37.4850	37.5564	37.6279	37.6995	37.7711
106	38.2024	38.2745	38.3466	38.4189	38.4912
107	38.9266	38.9993	39.0722	39.1451	39.2181
108	39.6576	39.7310	39.8046	39.8782	39.9519
109	40.3954	40.4695	40.5437	40.6180	40.6924
110	41.1400	41.2148	41.2897	41.3647	41.4397
111	41.8914	41.9669	42.0424	42.1181	42.1938
112	42.6496	42.7257	42.8020	42.8783	42.9547
113	43.4146	43.4914	43.5684	43.6454	43.7225
114	44.1864	44.2639	44.3415	44.4192	44.4970
115	44.9650	45.0432	45.1215	45.1999	45.2783
116	45.7504	45.8293	45.9082	45.9873	46.0664
117	46.5426	46.6221	46.7018	46.7815	46.8613
118	47.3416	47.4218	47.5022	47.5826	47.6631
119	48.1474	48.2283	48.3093	48.3904	48.4716
120	48.9600	. . .	. . .	. . .	. . .

The

The Areas of Circles in Wine Gallons.

Dia. in Inc.	.5	.6	.7	.8	.9
92	29.0912	29.1541	29.2171	29.2802	29.3433
93	29.7236	29.7872	29.8509	29.9146	29.9785
94	30.3628	30.4271	30.4915	30.5559	30.6204
95	31.0088	31.0738	31.1388	31.2039	31.2691
96	31.6616	31.7273	31.7930	31.8588	31.9246
97	32.3212	32.3875	32.4539	32.5204	32.5869
98	32.9876	33.0546	33.1217	33.1888	33.2561
99	33.6608	33.7285	33.7963	33.8641	33.9820
100	34.3408	34.4092	34.4776	34.5461	34.6147
101	35 0276	35.0967	35.1658	35.2350	35.3042
102	35.7212	35.7909	35.8607	35.9306	36.0005
103	36.4216	36.4920	36.5625	36.6330	36.7037
104	37.1288	37.1999	37.2711	37.3423	37.4136
105	37.8428	37.9146	37.9864	38.0583	38.1303
106	38.5636	38.6361	38.7086	38.7812	38.8538
107	39.2912	39 3043	39.4375	39.5108	39.5841
108	40 0256	40.0994	40.1733	40.2472	40.3212
109	40.7668	40.8413	40.9159	40.9905	41.0652
110	41 5148	41.5900	41.6652	41.7405	41.8159
111	42.2696	42.3455	42.4214	42.4974	42.5734
112	43.0312	43.1077	43.1843	43.2610	43.3377
113	43.7996	43.8708	43.9541	44.0314	44.1089
114	44.5748	44.6527	44.7307	44.8087	44.8868
115	45.3568	45.4354	45.5140	45.5927	45.6715
116	46.1456	46.2249	46.3042	46.3836	46.4630
117	46.9412	47.0211	47.1011	47.1812	47.2613
118	47.7436	47.8242	47.9049	47.9856	48.0665
119	48.5528	48.6341	48.7155	48.7969	48.8784

The Uſes of the preceding Tables of Areas are ſo very obvious, that we apprehend one Example will be ſufficient to illuſtrate them both. Let that be in finding the Area of a Circle, in Ale and Wine Gallons, whoſe Diameter is 45.4 Inches.

Againſt 45 in the 1ſt Column, under the Words *Diam. in Inches*, and in the 6th Column under .4, we have 5.7405 Gallons in the Table of Ale Areas; and 7.0079 Gallons, in the Table of Wine Areas, the Anſwer ſought.

If any One ſhould be inclined to proceed farther, with the foregoing Table of Ale Areas, the Method of Operation (by which the whole Table was computed) is as follows. — To .066814549 add .000055702, and the Sum .066870251 (called the *reſerved Sum)* being added to 40.10544, (the Area for 120 Inches Diameter) gives the Area for 120.1 Inches; again to .066870251 (the *reſerved Sum)* add the common Addend .000055702,* and this (*reſerved*) Sum .066925953, being added to 40.10544, gives 40.172365953 the Area for 120.2: Proceed in the ſame Manner, ſtill adding the laſt reſerved Sum and the common Addend (.000055702) together, and then adding the Sum of thoſe two, to the laſt Area, gives the ſucceeding Area; *i. e.* when the Diameter is increaſed by $\frac{1}{10}$th of an Inch.

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* The Reaſon of .000055702 being a common Addend for Ale, and .000068 a common Addend for Wine Gallons, is very evident from the *Lemma* in *Pa.* 146: For, in this Caſe, $n = .1$, $\therefore 2n^2 = \overline{.1}|^2 \times 2$ (or .02); which being multiplied by .0027851 for Ale, and .0034 for Wine (in Order to have its Meaſure in Parts of a Gallon) gives .000055702, the common Addend for Ale, and .00068, the common Addend for Wine Gallons, when the Diameter of the Circle is conſtantly increaſed by one-tenth of an Inch.

And for the Table of Wine Areas (which is derived from the very ſame Principle), proceed thus: —To .081566 add .000068, and this Sum .081634 (called the *reſerved Sum*) being added to 48.96 (the Area for 120 Inches Diameter) gives 49.041634, the Area for 120.1 Inches: Again to .081634, the laſt *reſerved Sum*, add the common Addend .000068, and we ſhall then get .081702 for the *reſerved Sum*; which being added to 49.041634 (found above) gives 49.123336, the Area for 120.2 Inches, and ſo on; ſee the following Operation.

Inches.		Gallons.		
120 - -		48.960000	0.081566	reſerved Sum at
	Add	.081634	.000068	[120 Inches.
120.1 -		49.041634	.081634	reſerved.
	Add	.081702	.000068	
120.2 -		49.123336	.081702	reſerved.
		.081770	.000068	
120.3 -		49.205106	.081770	
		.081838	.000068	
120.4 -		49.286944	.081838	
		.081906	.000068	
120.5 -		49.368850	.081906	
		.081974	.000068	
120.6 -		49.450824	.081974	
		&c.	&c.	

A TABLE *of the* Areas of the Segments of a Circle *whoſe Diameter is Unity, and ſuppoſed to be divided into* 1000 *equal Parts.*

Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*
.001	.000042	.030	.006865	.059	.018766
.002	.000119	.031	.007209	.060	.019239
.003	.000219	.032	.007558	.061	.019716
.004	.000337	.033	.007913	.062	.020196
.005	.000470	.034	.008273	.063	.020680
.006	.000618	.035	.008638	.064	.021168
.007	.000779	.036	.009008	.065	.021659
.008	.000951	.037	.009383	.066	.022154
.009	.001135	.038	.009763	.067	.022652
.010	.001329	.039	.010148	.068	.023154
.011	.001533	.040	.010537	.069	.023659
.012	.001746	.041	.010931	.070	.024168
.013	.001968	.042	.011330	.071	.024680
.014	.002199	.043	.011734	.072	.025195
.015	.002438	.044	.012142	.073	.025714
.016	.002685	.045	.012554	.074	.026236
.017	.002940	.046	.012971	.075	.026761
.018	.003202	.047	.013392	.076	.027289
.019	.003471	.048	.013818	.077	.027821
.020	.003748	.049	.014247	.078	.028356
.021	.004031	.050	.014681	.079	.028894
.022	.004322	.051	.015119	.080	.029435
.023	.004618	.052	.015561	.081	.029979
.024	.004921	.053	.016007	.082	.030526
.025	.005230	.054	.016457	.083	.031076
.026	.005546	.055	.016911	.084	.031629
.027	.005867	.056	.017369	.085	.032186
.028	.006194	.057	.017831	.086	.032745
.029	.006527	.058	.018296	.087	.033307

The Areas of the Segments of a Circle.

Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*
.088	.033872	.119	.052736	.150	.073874
.089	.034441	.120	.053385	.151	.074589
.090	.035011	.121	.054036	.152	.075306
.091	.035585	.122	.054689	.153	.076026
.092	.036162	.123	.055345	.154	.076747
.093	.036741	.124	.056003	.155	.077469
.094	.037323	.125	.056663	.156	.078194
.095	.037909	.126	.057326	.157	.078921
.096	.038496	.127	.057991	.158	.079649
.097	.039087	.128	.058658	.159	.080380
.098	.039680	.129	.059327	.160	.081112
.099	.040276	.130	.059999	.161	.081846
.100	.040875	.131	.060672	.162	.082582
.101	.041476	.132	.061348	.163	.083320
.102	.042080	.133	.062026	.164	.084059
.103	.042687	.134	062707	.165	.084801
.104	.043296	.135	.063389	.166	.085544
.105	.043908	.136	.064074	.167	.086289
.106	.044522	.137	.064760	.168	.087036
.107	.045139	.138	.065449	.169	.087785
.108	.045759	.139	.066140	.170	.088535
.109	.046381	.140	.066833	.171	.089287
.110	.047005	.141	.067528	.172	.090041
.111	.047632	.142	.068225	.173	.090797
.112	.048262	.143	.068924	.174	.091554
.113	.048894	.144	.069625	.175	.092313
.114	.049528	.145	.070328	.176	.093074
.115	.050165	.146	.071033	.177	.093836
.116	.050804	.147	.071741	.178	.094601
.117	.051446	.148	.072450	.179	.095366
.118	.052090	.149	.073161	.180	.096134

The Areas of the Segments of a Circle.

Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*
.181	.096903	.212	.121529	.243	.147512
.182	.097674	.213	.122347	.244	.148371
.183	.098447	.214	.123167	.245	.149230
.184	.099221	.215	.123988	.246	.150091
.185	.099997	.216	.124810	.247	.150953
.186	.100774	.217	.125634	.248	.151816
.187	.191553	.218	.126459	.249	.152680
.188	.102334	.219	.127285	.250	.153546
.189	.103116	.220	.128113	.251	.154412
.190	.103900	.221	.128942	.252	.155280
.191	.104685	.222	.129773	.253	.156149
.192	.105472	.223	.130605	.254	.157019
.193	.106261	.224	.131438	.255	157890
.194	.107051	.225	.132272	.256	.158762
.195	.107842	.226	.133108	.257	.159636
.196	.108636	.227	133945	.258	.160510
.197	.109430	.228	.134784	.259	.161386
.198	.110226	.229	.135624	.260	.162263
.199	.111024	.230	.136465	.261	.163140
.200	.111823	.231	.137307	.262	.164019
.201	.112624	.232	.138150	.263	.164899
.202	.113426	.233	.138995	.264	.165780
.203	.114230	.234	.139841	.265	.166663
.204	.115035	.235	.140688	.266	.167546
.205	.115842	.236	.141537	.267	.168430
.206	.116650	.237	.142387	.268	.169315
.207	.117460	.238	.143238	.269	.170202
.208	.118271	.239	.144091	.270	.171089
.209	.119083	.240	.144944	.271	.171978
.210	.119897	.241	.145799	.272	.172867
.211	.120712	.242	.146655	.273	.173758

The Areas of the Segments of a Circle.

Ver-ſed Sine	*Seg. Area*	Ver-ſed Sine	*Seg. Area*	Ver-ſed Sine	*Seg. Area*
.274	.174649	.305	.202761	.336	.231689
.275	.175542	.306	.203683	.337	.232634
.276	.176435	.307	.204605	.338	.233580
.277	.177330	.308	.205527	.339	.234526
.278	.178225	.309	.206451	.340	.235473
.279	.179122	.310	.207376	.341	.236421
.280	.180019	.311	.208301	.342	.237369
.281	.180918	.312	.209227	.343	.238318
.282	.181817	.313	.210154	.344	.239268
.283	.182718	.314	.211082	.345	.240218
.284	.183619	.315	.212011	.346	.241169
.285	.184521	.316	.212940	.347	.242121
.286	.185425	.317	.213871	.348	.243074
.287	.186329	.318	.214802	.349	.244026
.288	.187234	.319	.215733	.350	.244980
.289	.188140	.320	.216666	.351	.245934
.290	.189047	.321	.217599	.352	.246889
.291	.189955	.322	.218533	.353	.247845
.292	.190864	.323	.219468	.354	.248801
.293	.191775	.324	.220404	.355	.249757
.294	.192684	.325	.221340	.356	.250715
.295	.193596	.326	.222277	.357	.251673
.296	.194509	.327	.223215	.358	.252631
.297	.195422	.328	.224154	.359	.253590
.298	.196337	.329	.225093	.360	.254550
.299	.197252	.330	.226033	.361	.255510
.300	.198168	.331	.226974	.362	.256471
.301	.199085	.332	.227915	.363	.257433
.302	.200003	.333	.228858	.364	.258395
.303	.200922	.334	.229801	.365	.259357
.304	.201841	.335	.230745	.366	.260320

The

The Areas of the Segments of a Circle.

Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*
.367	.261284	.396	.289453	.425	.317981
.368	.262248	.397	.290432	.426	.318970
.369	.263213	.398	.291411	.427	.319959
.370	.264178	.399	.292390	.428	.320948
.371	.265144	.400	.293369	.429	.321938
.372	.266111	.401	.294349	.430	.322928
.373	.267078	.402	.295330	.431	.323918
.374	.268045	.403	.296311	.432	.324909
.375	.269013	.404	.297292	.433	.325900
.376	.269982	.405	.298273	.434	.326892
.377	.270951	.406	.299255	.435	.327882
.378	.271920	.407	.300238	.436	.328874
.379	.272890	.408	.301220	.437	.329866
.380	.273861	.409	.302203	.438	.330858
.381	.274832	.410	.303187	.439	.331850
.382	.275803	.411	.304171	.440	.332843
.383	.276775	.412	.305155	.441	.333836
.384	.277748	.413	.306140	.442	.334829
.385	.278721	.414	.307125	.443	.335822
.386	.279694	.415	.308110	.444	.336816
.387	.280668	.416	.309095	.445	.337810
.388	.281642	.417	.310081	.446	.338804
.389	.282617	.418	.311068	.447	.339798
.390	.283592	.419	.312054	.448	.340793
.391	.284568	.420	.313041	.449	.341787
.392	.285544	.421	.314029	.450	.342782
.393	.286521	.422	.315016	.451	.343777
.394	.287498	.423	.316004	.452	.344772
.395	.288476	.424	.316992	.453	.345768

The Areas of the Segments of a Circle.

Verſed Sine	*Seg. Area*	Verſed Sine	*Seg. Area*
.454	.346764	.478	.370706
.455	.347759	.479	.371705
.456	.348755	.480	.372704
.457	.349752	.481	.373703
.458	.350748	.482	.374702
.459	.351745	.483	.375702
.460	.352742	.484	.376702
.461	.353739	.485	.377701
.462	.354736	.486	.378701
.463	.355732	.487	.379700
.464	.356730	.488	.380700
.465	.357727	.489	.381699
.466	.358725	.490	.382699
.467	.359723	.491	.383699
.468	.360721	.492	.384699
.469	.361719	.493	.385699
.470	.362717	.494	.386699
.471	.363715	.495	.387699
.472	.364713	.496	.388699
.473	.365712	.497	.389699
.474	.366710	.498	.390699
.475	.367709	.499	.391699
.476	.368708	.500	.392699
.477	.369707		

The Uſe of the Line M.D *on the Sliding-Rule, in* Malt-Gauging.

It is preſumed that no great Difficulty can ariſe in the Practice of *Malt-Gauging*, if what has been delivered in *Sect*. VII. and VIII. be duly attended to: For it is well known, that a Malſter's *Ciſtern* (*viz.* where the Barley is ſteeped) and alſo the *Couch* (*i. e.* where it is laid after it has been ſteeped) are chiefly in the Form of a *Cylinder*, the *Fruſtum* of a *Cone*, or a *rectangular Parallelopepidon*; which Figures, with a Variety of others, have been fully treated of in the above-mentioned *Sections*, and their Contents computed, in Malt Buſhels, both by Pen and Sliding-Rule: However, it may not be amiſs to give a few Propoſitions, and exemplify the ſame, in Order to ſhew the Uſe of the Line M.D (commonly called *Malt-Depth*) on the Sliding-Rule.

Note. When the Barley is taken out of the Couch and ſpread on the Floor, it is then called a *Floor of Malt*.

Prop. I.

The Length and Breadth of a rectangular Parallelogram being given in Inches; to find the Area thereof (in Inches) *by the Line* M.D, &c. *on the Sliding-Rule.*

Rule.

To either of the given Dimenſions on M.D, ſet the *other* on the Line B, or, which is the very ſame (on ſome Rules), that marked N; then againſt 1 (*viz.* Unity) on M.D, is the required Area on the Slide.

Example.

Let the Length of a rectangular Parallelogram be 12, and the Breadth 7 Inches; required the Area thereof.

To 7 on M.D, ſet 12 on B (or N); then oppoſite 1 on M.D, is 84 on B, the Area ſought.*

Note. It is to be obſerved, in Examples of this Kind, that 1 on M.D muſt always repreſent Unity, and therefore the Factor taken on that Line, if greater than 21.5042, muſt be divided by ſuch a Power of 10 as will cauſe 1 on M.D to denote Unity; then the Number oppoſite thereto being multiplied by the ſame Power of 10 as the abovementioned Factor was divided by, and the Product will be the Anſwer ſought.

Prop. II.

The Length and Breadth of a rectangular Parallelogram being given in Inches; to find its Area in Malt Buſhels, by the Line M.D.

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* It is manifeſt, that, by ſetting 12 on B (or N) to 7 on M.D (or 7 on B to 12 on M.D), we ſhall obtain (on B) the Sum of the Diſtances of 1 to 12 on B, and 1 to 7 on M.D (or 1 to 7 on B, and 1 to 12 on M.D): But theſe Diſtances (by the Conſtruction of the Lines) are as the Logarithms of 7 and 12 reſpectively; conſequently the Sum of thoſe Diſtances will be as the Sum of the Logarithms of theſe Numbers, which, by the Property of Logarithms, is as the Logarithm of their Product.

RULE.

To either of the given Dimenſions on M.D, ſet the *other* on B (or N); then againſt 1 (*viz.* Unity) on A, is the required Area on B.

EXAMPLE.

Suppoſe the Length of a rectangular Ciſtern is 180, and the Breadth 53.5 Inches; required the Area thereof in Malt Buſhels.

As it is ſometimes difficult to eſtimate the *true* Value of the Number found upon the Line B; it may therefore be proper to lay down the following Directions.

Let 1, near the Middle of M.D, denote *Unity*; and the Number oppoſite thereto, at the Braſs Pin on A, repreſent 2150.42; then, in Order to have 1 at the Middle of the Line A to ſtand for *Unity* (inſtead of 1000), we need but to conceive the Product of the two given Factors to be divided by 1000:† Thus, in the Example before us, to 1.8

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† By ſuppoſing the Product of the two given Factors to be divided by 1000, is the very ſame Thing as ſuppoſing three Radii taken from the Lines M.D and B: For by ſetting 5.35 (inſtead of 53.5) on B to 1.8 (inſtead of 180) we ſhall obtain the *Diſtance* of 1 to 1.8 on M.D, and of 1 to 5.35 on B, in one Sum on B; which Diſtance is diminiſhed by that of 1 to 2.15042 (*i. e.* $\frac{2150.42}{1000}$) on A: Moreover, by the Conſtruction of the Lines, theſe Diſtances are as the Logarithms of the Numbers 1.8, 5.35, and 2.15042 reſpectively; whence, by the Properties of Logarithms, the Log. 1.8 + L. 5.35 − L. 2.15042 (= L. 180 + L. 53.5 − L. 2150.42 = L. 4.479) = $\frac{1.8 \times 5.35}{2.15042}$ (= $\frac{180 \times 53.5}{2150.42}$) = 4.479.

(inſtead of 180) on M.D, ſet 5.35 (inſtead of 53.5) on B; then againſt 1 on A is 4.5 Buſhels, *nearly*, on B.

Note. The Anſwer will come out the very ſame as above; if 1 at the Beginning of the Line A denotes *Unity*, the Number at the Braſs Pin, oppoſite 1 (*viz.* Unity) on M.D 215.042, and the Product of the two Factors on M.D and B be ſuppoſed to be divided by 100 (inſtead of 1000); but it will, I preſume, be better to keep to one *general Method*, as given above.

If the given Length of the Ciſtern is not leſs than 100 nor greater than 10000 (which laſt indeed never happens in Practice); then the required Area may be obtained, with more Eaſe to a Learner, by the Lines A and B. — Thus, in the laſt Example, to 2150 on A, ſet 53.5 on B; then oppoſite 180 (on the 1ſt Radius) on A, is 4.5 Buſhels, *nearly*, on B.

PROP. III.

The Length, Breadth, and Depth of a rectangular Parallelopipedon being given; to find its Content in Malt Buſhels, by the Line M.D, &c. *on the Sliding-Rule.*

RULE.

To any of the three given Dimenſions on M.D, ſet either of the other Two on B (N); then againſt the third Dimenſion on A, is the required Content on B.

EXAMPLE.

EXAMPLE.

Let the Length of a rectangular Floor of Malt be 350, the Breadth 160, and the Depth 6.5 Inches; required its Content in Malt Buſhels.

To 350 (or rather 3.5) on M.D, ſet 160 (or 16) on B (*vid.* the laſt *Example*); then againſt 6.5 (on the 2d Radius) on A, is 169 Buſhels on B.

Note. As 1 in the Middle of the Line A, according to our Method of Eſtimation, always denotes *Unity*, the third Dimenſion, when it exceeds 10, cannot be found on A: It will therefore be neceſſary, in ſuch Caſes, to have Recourſe to the Method laid down in *Pa.* 42.

Thus, for Inſtance, ſuppoſe the laſt Example had been a rectangular Ciſtern, whoſe Depth had been 65 Inches, and the other Dimenſions the ſame as before.

Then, the Rule being ſet as above, againſt 6.5 (*viz.* $\frac{1}{10}$th of 65) we have 169; which being multiplied by 10, gives 1690 Buſhels, the required Content of the Ciſtern, *nearly*.

The END.

To face the End.

Fig. I.

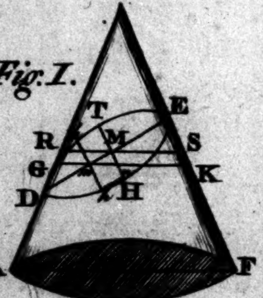


Fig. II.

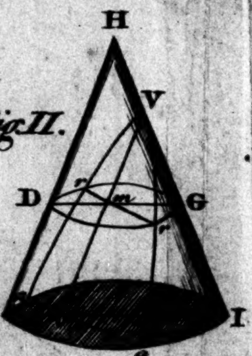
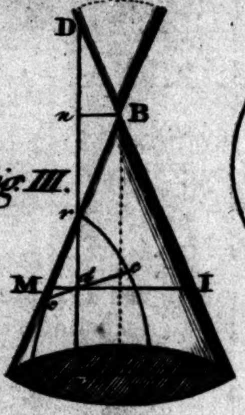


Fig. III.



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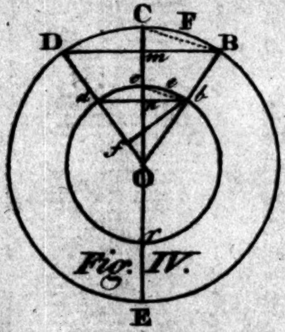

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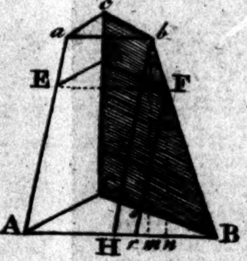

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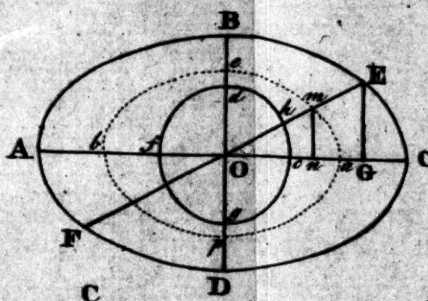

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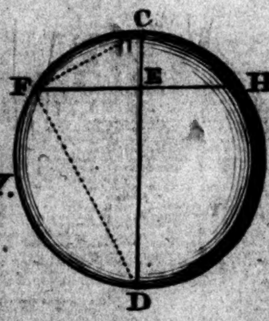

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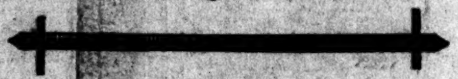

Fig. IX.

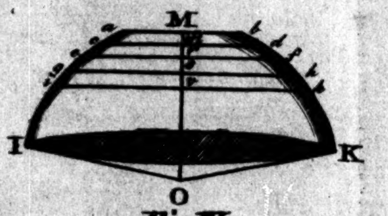

Fig. X.

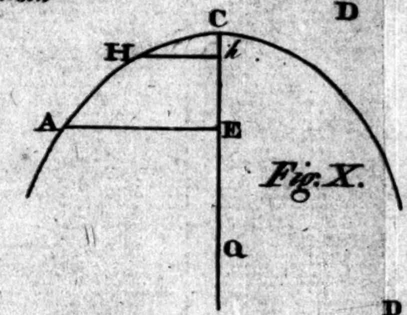

Fig. XI.

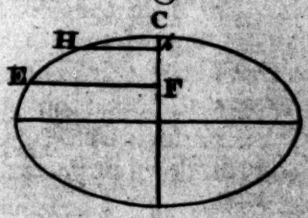

Fig. XII.

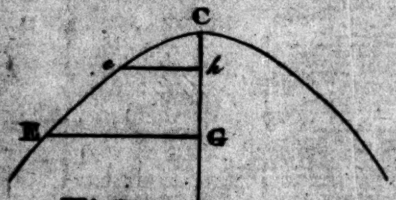

Fig. XIII.

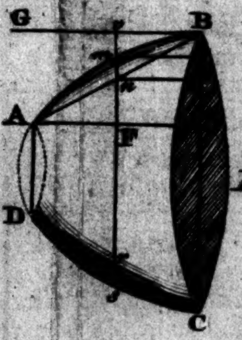

Fig. XVI.

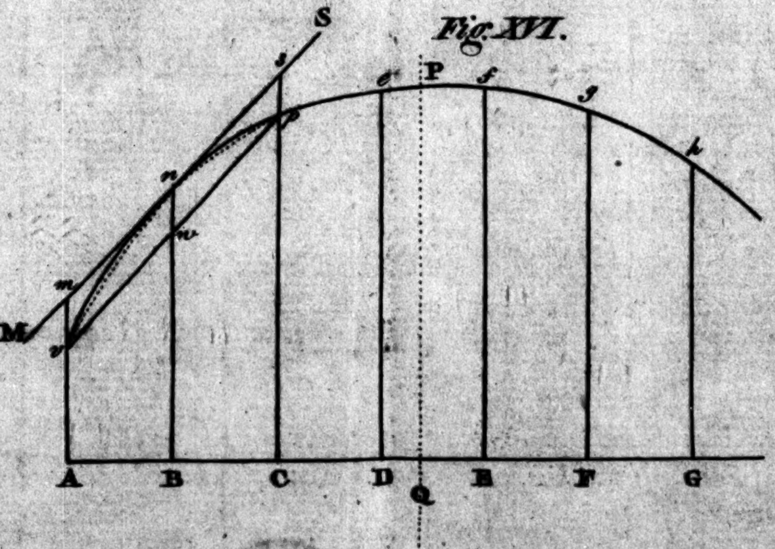

Fig. XVII.

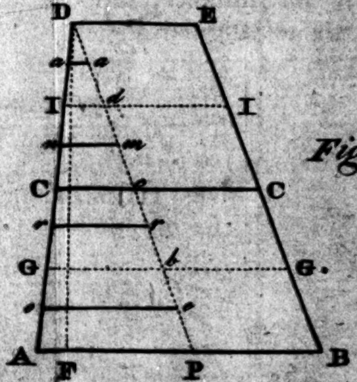

Fig. XIV.

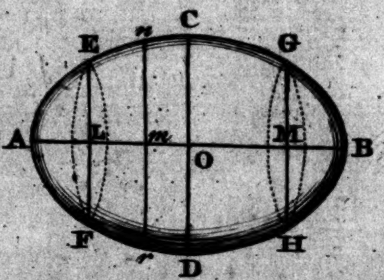

Fig. XV.